Q1. For the Mathieu equation
\[ \ddot{x} + (\delta + \epsilon \cos 2t) x = 0 \]
1. Show the transition curves near \( k = 1 \) are given by
\[ \delta = 1 \pm \frac{\epsilon}{2} - \frac{1}{32}\epsilon^2 + O(\epsilon^3) \]
2. Show that near \( k = 3 \) that both curves are given by
\[ \delta = 9 + \frac{\epsilon^2}{64} + \cdots \]

Q2. Go the following web site [http://monet.unibas.ch/~elmer/pendulum/vpend.htm](http://monet.unibas.ch/~elmer/pendulum/vpend.htm) and use the Java applet and do the suggested experiments. Now answer the following question

1. Is the pendulum stable if the frequency of the excitation is 1?
2. Is there a parameter range of amplitude and driving frequency were the upside pendulum is stable.
3. The \( \delta_0 = 0 \) transition curve exists for \( \epsilon < 0 \) by consider \( \cos(\theta + \pi) \) show this gives the stable region for the upside down pendulum.

Q3. For the equation
\[ \ddot{x} + (\delta + \epsilon \cos^3 t) x = 0 \]
1. Show that all the transition curves are given by
\[ \delta_0 = \frac{m^2}{4} \]
where \( m \) is an integer.
2. Show that near \( m = 0 \), the transitions curve is given by
\[ \delta = -\frac{41}{144}\epsilon^2 \cdots \]
3. Show that near \( m = 2 \) the curves are given by
\[ \delta = 1 + \frac{63}{320}\epsilon^2 + \cdots \quad \text{and} \quad \delta = 1 - \frac{7}{320}\epsilon^2 + \cdots \]