Week 19

1. Solve the following linear ODEs by finding an integrating factor:
   (a) \( \frac{dy}{dx} + y = \exp(-x) ; \ y = 2 \text{ when } x = 0. \)
   (b) \( \frac{dy}{dx} + y \cos x = \cos x ; \ y = 1 \text{ when } x = 0. \)
   (c) \( \frac{dy}{dx} + \frac{y}{x} = \sin x ; \ y = 0 \text{ when } x = 0. \) (Changed – it was \( y = 1 \) but that did not work!)

2. Let \( C \) be concentration of dissolved Oxygen in bioreactor and \( C_s \) concentration of dissolved Oxygen at saturation, and \( D = C_s - C \) the ‘deficit’. Let \( L \) be the constant Biological Oxygen Demand of organisms in the reactor. The following differential equation is given as a model
   \[ \frac{dD}{dt} = k_d L - k_r D \]
   where \( k_d \) and \( k_r \) are constants and \( t \) is time.
   (a) What does this equation mean?
   (b) Solve the differential equation, assuming \( D = D_0 \) at \( t = 0 \)
   (c) What shape is this curve?