LAPLACE TRANSFORMS

$$\tilde{f}(s) = \int_0^\infty e^{-st} f(t) dt$$

function	transform
1	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2}$
$\cosh \omega t$	$\frac{s}{s^2-\omega^2}$
$t \sin \omega t$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
$t \cos \omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
$H_a(t) = H(t-a)$	$\frac{e^{-as}}{s}$
$\delta(t)$	1
$e^{at} t^n$	$\frac{n!}{(s-a)^{n+1}}$
$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$
$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
$e^{at} \sinh \omega t$	$\frac{\omega}{(s-a)^2-\omega^2}$
$e^{at} \cosh \omega t$	$\frac{s-a}{(s-a)^2-\omega^2}$

Let $\tilde{f}(s) = \mathcal{L}\{f(t)\}$ then

$$\begin{aligned}
\mathcal{L}\left\{e^{at} f(t)\right\} &= \tilde{f}(s-a), \\
\mathcal{L}\{t f(t)\} &= -\frac{d}{ds}(\tilde{f}(s)), \\
\mathcal{L}\left\{\frac{f(t)}{t}\right\} &= \int_{x=s}^{\infty} \tilde{f}(x) dx \quad \text{if this exists.}
\end{aligned}$$

<u>Derivatives and integrals</u>

Let $y = y(t)$ and let $\tilde{y} = \mathcal{L}\{y(t)\}$ then

$$\begin{aligned}
\mathcal{L}\left\{\frac{dy}{dt}\right\} &= s\tilde{y} - y_0, \\
\mathcal{L}\left\{\frac{d^2y}{dt^2}\right\} &= s^2\tilde{y} - sy_0 - y_0', \\
\mathcal{L}\left\{\int_{\tau=0}^{t} y(\tau) d\tau\right\} &= \frac{1}{s}\tilde{y}
\end{aligned}$$

where y_0 and y_0' are the values of y and dy/dt respectively at $t = 0$.

<u>Time delay</u>

Let $$g(t) = H_a(t) f(t-a) = \begin{cases} 0 & t < a \\ f(t-a) & t > a \end{cases}$$

then $$\mathcal{L}\{g(t)\} = e^{-as}\tilde{f}(s).$$

<u>Scale change</u>

$$\mathcal{L}\{f(kt)\} = \frac{1}{k}\tilde{f}\left(\frac{s}{k}\right).$$

<u>Periodic functions</u>

Let $f(t)$ be of period T then

$$\mathcal{L}\{f(t)\} = \frac{1}{1 - e^{-sT}} \int_{t=0}^{T} e^{-st} f(t) dt.$$

Convolution

Let $f(t) * g(t) = \int_{x=0}^{t} f(x)g(t-x)dx = \int_{x=0}^{t} f(t-x)g(x)dx$

then $\mathcal{L}\{f(t) * g(t)\} = \tilde{f}(s)\tilde{g}(s).$

RLC circuit

For a simple RLC circuit with initial charge q_0 and initial current i_0,

$$\tilde{E} = \left(r + Ls + \frac{1}{Cs}\right)\tilde{i} - Li_0 + \frac{1}{Cs}q_0.$$

Limiting values

initial value theorem

$$\lim_{t\to 0+} f(t) = \lim_{s\to\infty} s\tilde{f}(s),$$

final value theorem

$$\begin{aligned} \lim_{t\to\infty} f(t) &= \lim_{s\to 0+} s\tilde{f}(s), \\ \int_0^\infty f(t)dt &= \lim_{s\to 0+} \tilde{f}(s) \end{aligned}$$

provided these limits exist.