# SVD analysis of GPR full-waveform inversion

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To clear the 110 million active landmines in place using current technologies would cost an estimated $30 billion and take over 1000 years. Detection methods have hardly changed since World War II, and are largely based on simple metal decetion. The process is extremely slow due to large amounts of clutter, and almost every piece of detected metal needing to be carefully removed. Adding a second detection or imaging modality, such as ground penetrating radar (GPR), can significantly speed the clearance process by giving demining personnel more information to distinguish clutter from landmines.

For GPR, we pose the task of imaging the subsurface as full-waveform inversion (FWI). That is to solve the inverse problem

$$\mathbf{m}_{\mathrm{im}} = \underset{\mathbf{m}}{\operatorname{argmin}} \|\mathcal{F}[\mathbf{m}] - \mathbf{d}\|^2 + \lambda R(\mathbf{m}), \tag{1}$$

where $\mathbf{d}$ is the recorded GPR data, $\mathbf{m}$ is a set of parameters describing the subsurface (such as dielectric permittivity), $\mathcal{F}$ is the forward operator simulating GPR data, $\lambda > 0$ and $R$ is a regularisation term which incorporates a-priori knowledge and prevents over-solving. FWI naturally encorporates multiple scattering from ground clutter into the imaging procedure and updates the wave propagation characteristics of the background medium (which aren't known exactly). It also provides quantitative information (as well as qualitative), which could help deminers better classify detected objects.

Working alongside the development of GPR equipment, it is beneficial to determine what the minimum requirements of acquisition system are to be able to perform FWI, and what form of equipment will give best results. Particularly, we are interested in whether an affordable bi-static system (with one source and one receive antenna) is sufficient, or if something more complex is necessary.

To help us answer this question we perform an SVD analysis of the Jacobian matrix $J$ of the first (data misfit) term in (1), asserting that this matrix exhibits the dominant features of the map from data to image. While the right singular vectors of $J$ span the whole image space, taking only those which correspond to singular values above the noise level of the data leaves us with a large nullspace. Projecting onto these first singular vectors gives us a way of comparing the ability of acquisition systems to resolve objects of interest, which we can verify by solving the full-wave problem (1).