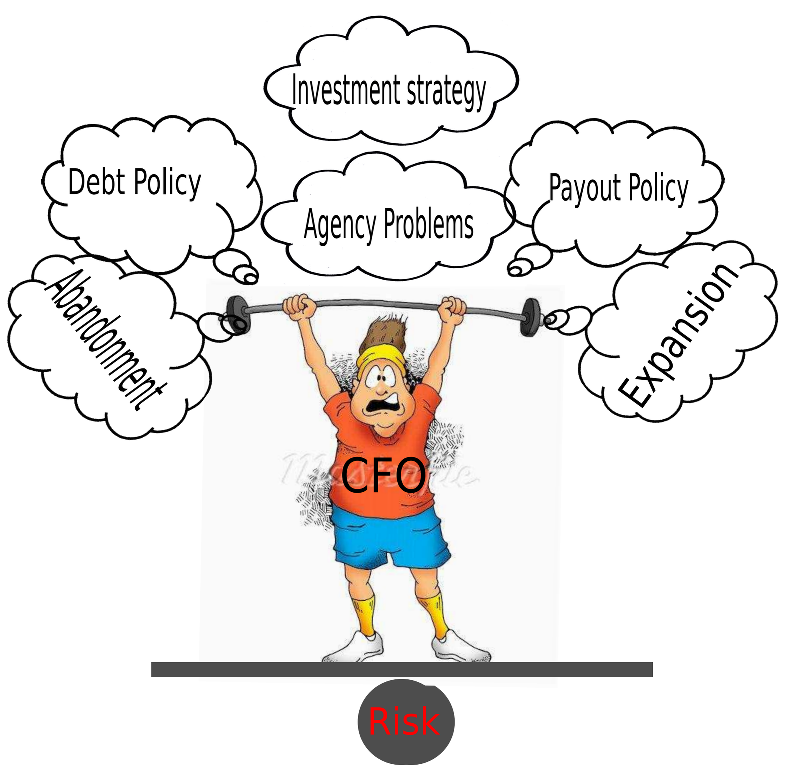
MANCHESTER 1824

The University of Manchester

The Optionality of a Financial Constrained Firm

Mingliang Cheng, Geoffrey W.Evatt, Paul Johnson & Peter Duck

Struggling Chief Financial Officer:



The increasing complexity of modern corporate finance problems are plaguing decision makers. For example, how should one balance and prioritise investments decisions with dividend payouts and debts policies? Classic theories are good at solving each of those problems in isolation, but has difficulty in considering them jointly owing to a lack of proper models. Thus, a unifying and consistent method that can combine multiple decisions is highly desirable. In this poster, we build a model of a financial constrained firm that is exposed to uncertainty in their revenue. In doing so, we combine classic Real Option techniques with Probability Theory, capital structure and balance sheet considerations, so as to optimise equity value. In this way, we give actionable solutions to guide the firms' complex financial decisions.

Mathematical Model:

Consider a generic **Cash-Holding-Firm** that is exposed to uncertainties in the size of their future revenue (R). Cash holding level is time based and denoted as C. The firms objective is to maximise shareholder's value (V) by appropriately controlling abandonment, dividend payments (d) and borrowings (C_b). Assuming R follows a stochastic process and cash holdings is path dependent on R. We have,

$$dR = \mu(t)Rdt + \sigma(t)RdW_t \qquad (1) \qquad\qquad dC = \big(R - \varepsilon - d\big)dt \qquad (2)$$

Thus, shareholders' value can be generated by the expected accumulated future dividends,

$$V = \max_d\{E[\int_0^\tau de^{-rt}dt]\}. \qquad (3)$$

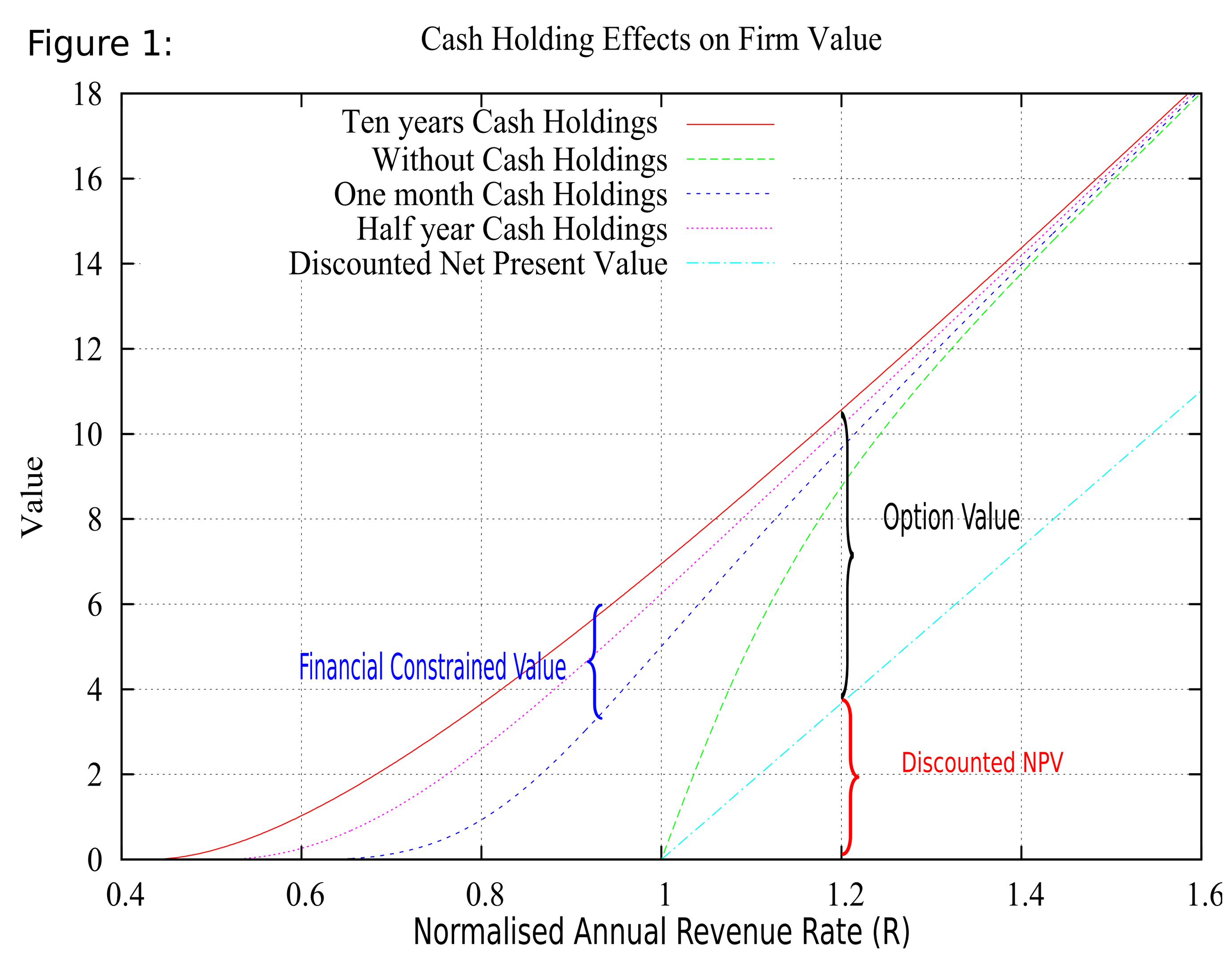
According to Hamilton-Jacobi-Bellman equation, we can model the system as,

$$\frac{1}{2}\sigma^2R^2\frac{\partial^2 V}{\partial R^2} + \mu R\frac{\partial V}{\partial R} - rV + \max_d\left\{(R + r_cC - \varepsilon - d)\frac{\partial V}{\partial C} + d\right\} + \frac{\partial V}{\partial t} = 0. \qquad (4)$$

$$\begin{cases} V = C \ \text{ as } \ \tau = T \\ \frac{\partial^2 V}{\partial R^2} = 0 \ \text{ as } \ R \to \infty \\ \frac{\partial V}{\partial C} = 0 \ \text{ as } \ C \to \infty \ \& \ R + r_cC - \varepsilon - d > 0 \\ V = E_a \ \text{ as } \ C = 0 \ \& \ R + r_cC - \varepsilon - d \leq 0 \\ V = E_a \ \text{ as } \ R \leq R^* \end{cases}$$

Overvalued Real Option:

As shown in these equations, our model differs from classic option studies primarily by the inclusion of variable in C, which influences the degree to which the model is financially constrained.



In this figure, we compare three different solutions: the **Net Present Value**, the **Classic Real Options valuation** and our **Financial Constrained valuation**. The results show why a project can be undervalued if Net Present Value Method is used, and how much it is overvalued by using classic Real Options approach. This is because NPV ignores all value of flexible management. At the other extreme, the classic Real Options approach assumes no monetary constraints and thus assumes perfect flexibility at all times. Our new approach accounts for all potential scenarios of the firm's financial constraints, and thus presents a more realistic picture of the firms' shareholder value.

Dividends Policy:

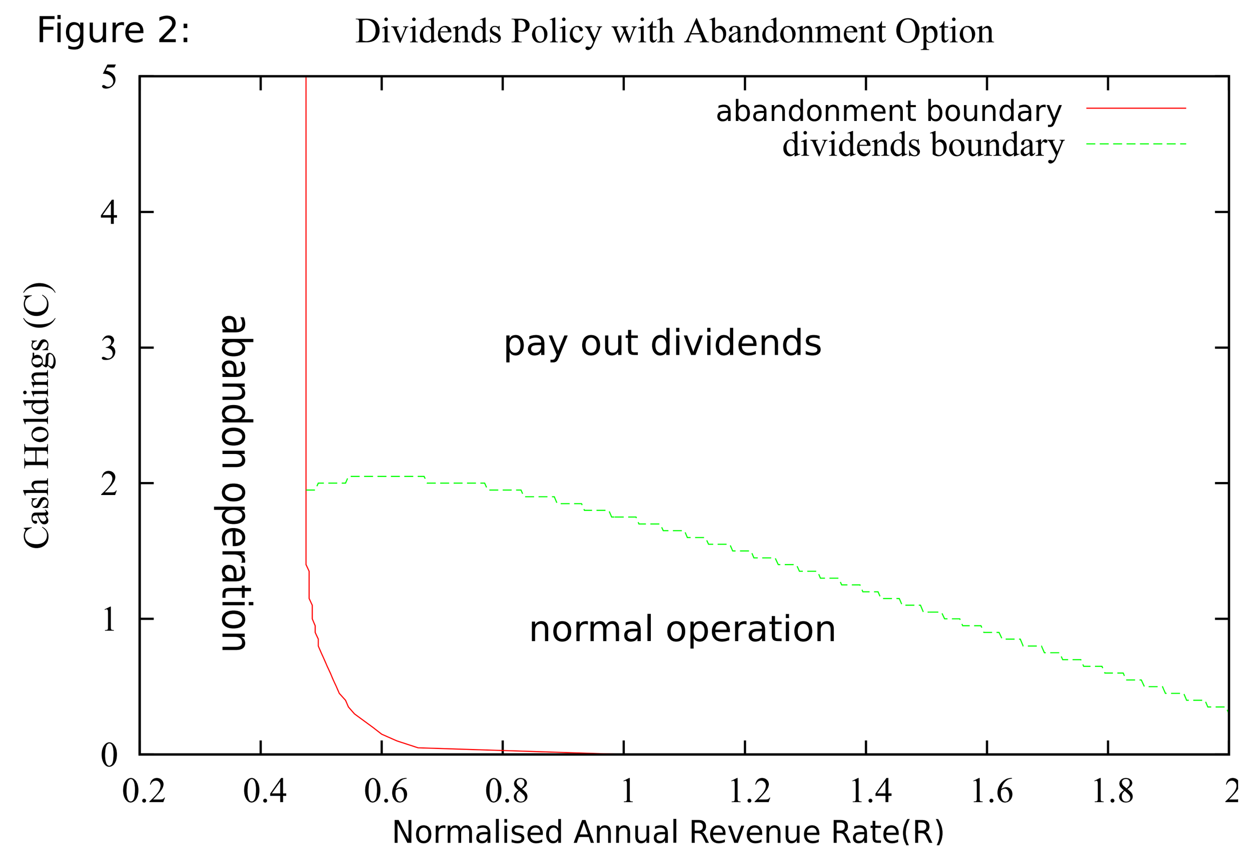
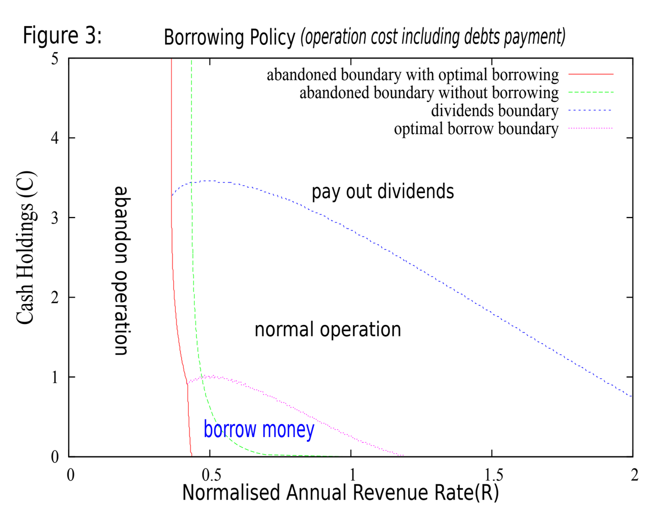


Figure 2 shows two boundaries of three different running strategies: Abandon operation, normal operation and pay out dividends. For the abandonment boundary, it changes significantly at low levels of cash holdings (enough for 1-2 years operation) and turns to be flat when the firm has superfluous cash holdings (over 3 years operation costs). This shows how cash holdings prevent short term running risks. According to the dividends boundary, financial managers should make dividends policy depends not only on the firm's liquidity but also on the firm's profitability. Generally, high profitability corresponds to positive dividends policy.

Debts Policy:



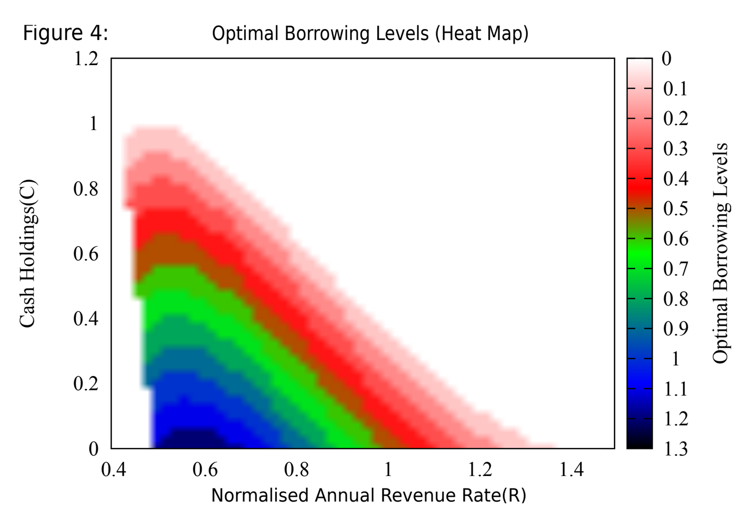


Figure 3 and 4 show firm optimal borrowing strategies with different operation state.
As shown in figure 3, three boundaries (abandoned boundary, borrowing boundary and dividends boundary) divide the whole region into four different parts. The small part at the bottom is the region where firm should borrow money from bank. Comparing abandonment boundaries with the optimal borrowing policy (red line) and that without using optimal borrowing policy (green line), we can see the abandonment boundary (with optimal borrowing policy) has moved to the left, especially in the region around ($R = 1, C < 1$). This is because optimal borrowing helps the firm get rid of risk when it doesn't have enough cash holdings.
In figure 4, the heat map shows how much money should the firm borrow at each finance state. The optimal borrowing level changes from 0 to 2.2 and decreases with the increasing of cash holdings.

Discussion and Conclusion:

In this poster, we modelled a financial constrained firm by extending the dimension of the Cash Holdings. By using this model, we show the deficiencies of NPV and classic Real Options analysis and explain further why classic Real Option generally overvalue a project. Based on this, we show a unifying and consistent method to combine the investment strategy, dividends policy and debts problem together within the given model structure. These contributions can help managers make more accurate and practical decisions in handling capital budgeting decisions. Of course, there are a lot of aspects of the model we could improve: for example, optimal lending interest rate(in view of (commercial bank), optimal financing (internal finance, issuing shares and bounds), we aim to include these in future work.

References:

- Evatt, G.W., Johnson, P., Cheng, M., Glover, K., 2012. Optimal Bank and Regulatory Capital Reserve Strategies Under Loan-Loss Uncertainty, SSRN 2136977
- Brealey, R.A. and Myers, S.C. and Allen, F., Principles of corporate finance,10th edition, 2011, McGraw-Hill
- Johnathan, M. Real option analysis(tools and techniques for valuing strategic investments and decisions), 2002, John Wiley & Sons, Inc.