MATHx6101 Numerical Linear Algebra: Syllabus 2020

Credit Rating	15
Level	Level 4/Postgraduate
Delivery	Semester 1
Coordinators	Professor N. J. Higham, Alan Turing 2.110, tel. 55822, `nick.higham@manchester.ac.uk` Professor F. Tisseur, Alan Turing 2.108, tel. 55823, `francoise.tisseur@manchester.ac.uk`

General description

This module treats the main classes of problems in numerical linear algebra: linear systems, least squares problems, and eigenvalue problems, covering both dense and sparse matrices. It provides analysis of the problems along with algorithms for their solution. It also uses MATLAB as tool for expressing and implementing algorithms and describes some of the key ideas used in developing high-performance linear algebra codes. Applications, such as machine learning and search engines, will be introduced throughout the module.

Aims

To develop understanding of modern methods of numerical linear algebra for solving linear systems, least squares problems, and the eigenvalue problem.

Learning Outcomes

On completion of the module, students will be able to

1. construct some key matrix factorizations using elementary transformations,
2. choose an appropriate numerical method to solve linear systems, least squares problems, and the eigenvalue problem,
3. evaluate and compare the efficiency and numerical stability of different algorithms for solving linear systems, least squares problems, and the eigenvalue problem,
4. design algorithms that exploit matrix structures such as triangularity, sparsity, banded structure, and symmetric positive definiteness,
5. quantify the sensitivity of a linear system or least squares problem to perturbations in the data.

Syllabus

1. **Basics.** Summary/recap of basic concepts from linear algebra and numerical analysis: matrices, operation counts. Matrix multiplication, block matrices. 4

Matrix norms. Linear system sensitivity. 2

2. **Matrix factorizations.** Cholesky factorization. QR factorization by Householder matrices and by Givens rotations. 3

LU factorization and Gaussian elimination; partial pivoting. Solving triangular systems by substitution. Solving full systems by factorization. Error analysis. Complete pivoting, rook pivoting. Numerical examples. 4

4. **Sparse and banded linear systems** Storage schemes for banded and sparse matrices. LU factorization, Markowitz pivoting. 2

5. **Linear least squares problem.** Basic theory using singular value decomposition (SVD) and pseudoinverse. Perturbation theory. Numerical solution: normal equations. SVD and rank deficiency. 3

6. **Iterative methods for linear systems** Iterative methods: Jacobi, Gauss-Seidel and SOR iterations. Kronecker product. Krylov subspace methods, conjugate gradient method. Preconditioning. 4

7. **Eigenvalue problem.** Basic theory, including perturbation results. Power method, inverse iteration. Similarity reduction. QR algorithm. 5

Textbooks

[1] Timothy A. Davis. *Direct Methods for Sparse Linear Systems*. Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 2006. ISBN 0-89871-613-6. xii+217 pp.

[2] James W. Demmel. *Applied Numerical Linear Algebra*. Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 1997. ISBN 0-89871-389-7. xi+419 pp.

[3] Gene H. Golub and Charles F. Van Loan. *Matrix Computations*. Johns Hopkins University Press, Baltimore, MD, USA, fourth edition, 2013. ISBN 978-1-4214-0794-4. xxi+756 pp.

[4] Desmond J. Higham and Nicholas J. Higham. *MATLAB Guide*. Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, third edition, 2017. ISBN 978-1-61197-465-2. xxvi+476 pp.

[5] Nicholas J. Higham. *Accuracy and Stability of Numerical Algorithms*. Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, second edition, 2002. ISBN 0-89871-521-0. xxx+680 pp.

[6] Yousef Saad. *Iterative Methods for Sparse Linear Systems*. Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, second edition, 2003. ISBN 0-89871-534-2. xviii+528 pp.

[7] G. W. Stewart. *Introduction to Matrix Computations*. Academic Press, New York, 1973. ISBN 0-12-670350-7. xiii+441 pp.

[8] G. W. Stewart. *Matrix Algorithms. Volume I: Basic Decompositions*. Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 1998. ISBN 0-89871-414-1. xx+458 pp.

[9] G. W. Stewart. *Matrix Algorithms. Volume II: Eigensystems*. Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 2001. ISBN 0-89871-503-2. xix+469 pp.

[10] Lloyd N. Trefethen and David Bau III. *Numerical Linear Algebra*. Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 1997. ISBN 0-89871-361-7. xii+361 pp.

[11] David S. Watkins. *Fundamentals of Matrix Computations*. Wiley, New York, third edition, 2010. ISBN 978-0-470-52833-4. xvi+644 pp.

Teaching and Learning Methods

30 lectures (two or three lectures per week), with a fortnightly examples class.
Special arrangements for 2020 to be advised.

Assessment

An end of module three hour examination (80%) and coursework (20%) comprising one assignment.
Special arrangements for 2020 to be advised.

Learning Hours

Activity	Hours
Staff/student contact	42
Work on assessments	40
Private study	68
Total	150