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Book Reviews

Edited by R. B. Kellogg

Featured Review: Selected Books on Numerical Linear Algebra

Numerical Linear Algebra and Applications. *By Biswa Nath Datta.* Brooks/Cole, Pacific Grove, CA, 1995. $83.95. xxii+680 pp., hardcover. ISBN 0-534-17466-3.

Applied Numerical Linear Algebra. *By James W. Demmel.* SIAM, Philadelphia, PA, 1997. $45.00. xi+419 pp., softcover. ISBN 0-89871-389-7.

Numerical Linear Algebra. *By Lloyd N. Trefethen and David Bau* III. SIAM, Philadelphia, PA, 1997. $34.50. xii+361 pp., softcover. ISBN 0-89871-361-7.

The world of numerical linear algebra has always been well endowed with outstanding books. Wilkinson's classic monograph *The Algebraic Eigenvalue Problem* [13] is the best-known book in the area from the 1960s and is still used today. Forsythe and Moler's *Computer Solution of Linear Algebraic Systems* (1967) [2] broke new ground by including listings of computer programs (in several languages) for solving linear systems by Gaussian elimination. In *Solving Least Squares Problems* (1974) [6], Lawson and Hanson gave the first comprehensive treatment of linear least squares problems. The most successful early general textbook was Stewart's *Introduction to Matrix Computations* (1973) [8], which was notable for giving a thorough and insightful treatment of the QR algorithm for eigenproblems. An influential monograph on the symmetric eigenproblem is Parlett's *The Symmetric Eigenvalue Problem* (1980) [7]. The current "bible" of numerical linear algebra, Golub and Van Loan's *Matrix Computations* (1996) [3], is now in its third edition, having first been published in 1983. All these books have given me great enjoyment. I learned numerical linear algebra from Stewart's book as an undergraduate, spent a summer reading Parlett prior to beginning graduate work, and, as an M.Sc. student, read one of the first copies of Golub and Van Loan to reach the UK. These and other books have set a high standard of exposition for later authors in the area to live up to.

Since the early 1980s books in numerical linear algebra have been published at a growing pace. For this review I have chosen three of the most recent textbooks: Datta (1995), Demmel (1997), and Trefethen and Bau (1997). Notable earlier textbooks include Hager (1988) [4], Strang (1988) [10], and Watkins (1991) [11]; my own contribution is [5]; and the first volume of Stewart's *Matrix Algorithms* [9] appeared as this review was being written. Datta is aimed at undergraduate and beginning graduate level courses, while Demmel and Trefethen and Bau target graduate level courses. All three books have been extensively classroom tested. All give MATLAB exercises, and Datta and Demmel provide MATLAB software that is used in the exercises and is available over the Web (from ftp://ftp.mathworks.com/pub/books/datta/ and http://www.siam.org/books/demmel/demmel_class, respectively).

All the books cover linear systems (solution by direct and iterative methods), the least squares problem, eigenvalue and singular value problems, and aspects of numer-

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ical stability. I now examine the books in more detail, pointing out the distinctive features of each.

Datta's book has two main strengths. First, the author goes out of his way to make the text easy to read, giving detailed derivations and carefully introducing and summarizing each chapter. An appendix offers a useful glossary of terms in numerical linear algebra. A section entitled "Suggestions for Further Reading" at the end of each chapter gives pointers to further information and recent developments; they include some interesting references of which I was unaware. Many numerical examples are given, although how useful these are when a student has access to MATLAB and so can carry out his or her own experiments is debatable. The second strength of Datta's book is the applications presented toward the beginning of most of the later chapters. Chapter 9 on the generalized eigenvalue problem is particularly strong in this respect, describing applications to vibrations of structures and model reduction, including the effect of earthquakes on a building. This chapter, which includes the quadratic eigenvalue problem, contains the best textbook presentation of the generalized eigenvalue problem that I have seen.

My criticisms of Datta's book are that the writing sometimes lacks precision, there are some dubious statements, and some explanations are sketchy. A few examples suffice to illustrate. Definition 3.3.1 of an ill-conditioned problem states that a small relative change in the data produces a large relative change in the solution; this would be correct if *produces* was replaced by *can produce*. Bunch's notions of strong and weak stability [1] are presented in section 3.8, where it is stated that "Weak stability is good enough for most users." I disagree. An algorithm for solving a square linear system $Ax = b$ for which the forward error behaves roughly like $\kappa(A)^4$ times the machine precision, where the condition number $\kappa(A) = \|A\|\|A^{-1}\|$, would be weakly stable (as it solves well-conditioned problems accurately) but of little practical use. In section 2.7 the reader is advised to accumulate inner products in extended precision during the computation of matrix-vector and matrix-matrix products, advice that was current 30 years ago but that is not appropriate for general users today. In presenting the QR algorithm for the nonsymmetric eigenproblem a standard deflation test for the Hessenberg iterates is stated, but the usual justification based on backward error is omitted; oddly, the reader is referred to Parlett's book on the *symmetric* eigenproblem [7].

Unlike most current textbooks, Datta uses the definition of flop (floating point operation) given in the first edition of [3], which is roughly half a new flop. For compatibility with current textbooks and research papers it would be desirable to convert to new flops for a subsequent edition. Unusually and unfortunately, throughout the book equations are not punctuated.

Demmel's book is the most ambitious of the three in terms of content and demands the most from the reader. In addition to standard material it includes sparse direct methods for linear systems, relative perturbation theory for the eigenvalue and singular value problems (the subject of much research in the last 10 years), Toda lattices, the fast Fourier transform, block cyclic reduction, and multigrid and domain decomposition techniques. The reader is brought to a stage where many of the current best algorithms and their pros and cons can be appreciated. In particular, a detailed comparison of methods for solving the symmetric eigenproblem is given, illustrated with color graphs of performance. A large set of exercises of varying, labeled difficulty is provided, which will be a boon to instructors. This is the only book of the three to describe the use of blocking and basic linear algebra subprograms (BLAS) to obtain increased performance on high-performance computers. In the preface Demmel states that he aimed to produce a practically oriented, self-contained book that teaches mathematics as well as software issues, all fitting within one semester. Except for the last aim, he has succeeded. I found a few minor errors (for example, missing nonsingularity assumptions on X and Y in Theorem 5.6 and Corollary 5.2, missing n dependence in the bound of Theorem 5.15), but these will no doubt be corrected in later printings. As another reviewer has noted [12], navigating the book is complicated by the fact that theorems, lemmas, algorithms, and so on are all numbered in different sequences. Given that the book is typeset in LaTeX, it would be trivial to change to a single numbering sequence in a future edition.

Trefethen and Bau differs from the other two books in that each of the 40 chapters is designed to occupy one lecture, which breaks the material into easily digestible pieces. This is the most polished of the three books—exquisitely written with barely a typo to be found. My overriding impression is of a book that brings fresh new ideas to the teaching of numerical linear algebra. Rather than start with Gaussian elimination, the authors first treat QR factorization, which provides a more unifying connection between the different algorithms treated in the book. This approach helps to maintain students' interest, as they are unlikely to have previously studied QR factorization in great detail. A further benefit is that there are at least four mathematically different ways of computing a QR factorization (classical and modified Gram–Schmidt, Householder transformations, and Givens transformations), so QR provides an excellent example for the comparison of competing methods. In Lecture 10 the difference between the Gram–Schmidt method and Householder QR factorization is boiled down to

Gram–Schmidt:	triangular orthogonalization,
Householder:	orthogonal triangularization.

Gems of observation such as this are invaluable for getting across subtle ideas to students. Other notable features of the book include a strong treatment of Krylov subspace methods for linear equation and eigenvalue problems, extensive use of explanatory diagrams, a nonstandard and intriguing definition of numerical instability, exercises at the end of each lecture, and illuminating notes on each lecture at the end of the book. Unlike in Demmel or Datta, matrices are assumed to be complex for most of the book. My preference in teaching numerical linear algebra is to work over the reals where possible, to help the weaker students and to avoid distractions such as the greater freedom over the choice of sign in a Householder vector.

Here, then, are three books all of which have received enthusiastic reviews and are currently being used as course texts at different institutions around the world (as searches on the World Wide Web readily reveal). Which to choose for teaching depends on the aims, level, and emphasis of the course. Personally, I am glad to have all three on my bookshelf as teaching resources, with Demmel's book being useful for my research, too.

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Ordinary Differential Equations. *By Wolfgang Walter.* Springer-Verlag, New York, 1997. $49.95. viii+380 pp., hardcover. ISBN 0-387-98459-3.

The present book is the outgrowth of the author's book on Differentialgleichungen (ordinary differential equations), which was published in 1972. It is based on a translation of the latest edition, which appeared in 1996, and deals with some important topics that were not treated there.

The book is a concise and thoroughly readable treatment of the subject aimed at students who are comfortable with the fundamentals of real analysis and linear algebra. For complex differential equations, some knowledge of holomorphic functions and their integrals is required. This and the tools needed from functional analysis, such as the contraction principle, Schauder's fixed point theorem, and Lebesgue's theory of integration, are described in the appendix and sections entitled"Supplements."

One of the very important features of the book that sets it apart from other texts is the fact that it contains material that is rarely found in textbooks. More important, the author provides new proofs for basic theorems. In this regard functional analysis and differential inequalities play a vital role throughout the book.

The text starts with an introductory chapter on classical cases of first-order equations. Through a number of examples, the reader is introduced to important features of initial value problems such as uniqueness and nonuniqueness, maximal solutions (in the case of nonuniqueness), and continuous dependence on initial data in the small. The phase plane and phase portraits are also touched upon.

Chapter II begins with the exposition of the theory. This chapter and the following one treat the initial value problem first for a single equation and then for systems of equations. The complex case where the solutions are holomorphic is discussed, and the theory of differential inequalities in one dimension is presented. An extension to differential inequalities in n dimensions is described in Supplement I of Chapter III.