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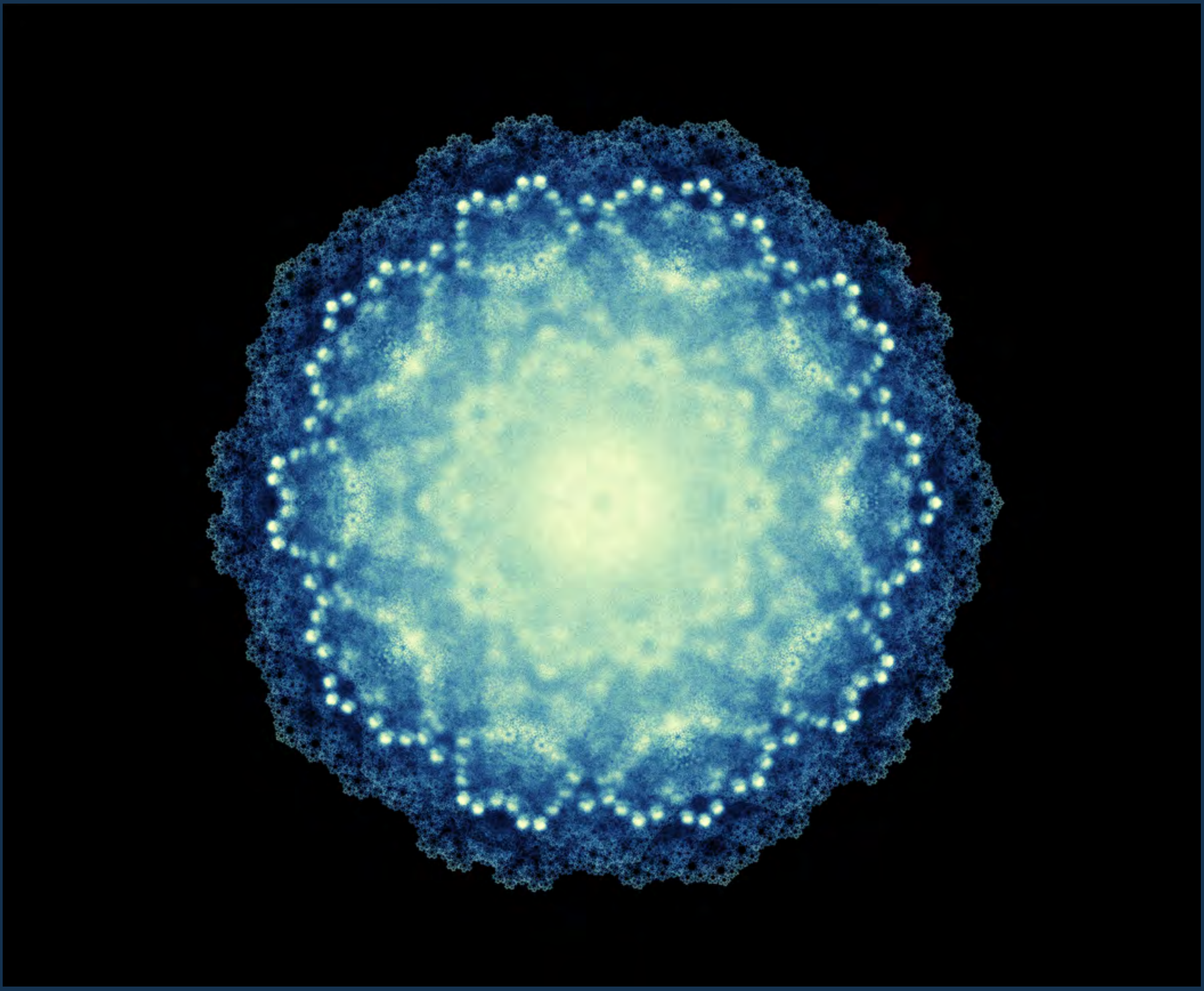

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THE PROBABILIST'S URN

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COVER IMAGE

See the article 'Bohemian Matrices' on page 16.

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# CONTENTS

# Cover Image: Bohemian Matrices

R.M. CORLESS, N.J. HIGHAM AND S.E. THORNTON

A family of Bohemian matrices is a set of matrices in which the matrix entries belong to a discrete set of numbers bounded independently of the matrix dimensions. The term is a contraction of BOunded HEight Matrix of Integers. Such matrices arise in many applications, and include $\{0,1\}$ graph incidence matrices and $\{-1,1\}$ Bernoulli matrices.

Many interesting and often easy-to-pose questions arise in connection with a given $n \times n$ Bohemian matrix family $\mathcal{B}$, either for general $n$ or for a particular $n$, including: what fraction of matrices in $\mathcal{B}$ are singular?; what are the most ill conditioned matrices in $\mathcal{B}$?; and how many distinct characteristic polynomials are there for matrices in $\mathcal{B}$?

The study of Bohemian matrices is not new; Taussky and others were considering them sixty years ago. Today, though, the computational power available at our fingertips can provide illumination in various ways, including through brute force calculations for small dimensions, through solving carefully posed optimization problems, and through visualising various quantities of interest.

Our example visualisation on the front cover of this *Newsletter* is a density plot over the complex plane of the eigenvalues of $10$ million $13 \times 13$ upper Hessenberg Toeplitz matrices with diagonal entries fixed at $0$, subdiagonal entries fixed at $1$, and all other entries sampled randomly from the set of fifth roots of unity. The image raises several questions, in particular about how the restriction to upper Hessenberg Toeplitz structure affects the distribution of the eigenvalues as the dimension $n$ goes to infinity.

For matrices $M_n$ from an *unstructured* matrix family with entries drawn independently and identically distributed from a population with zero mean and unit variance the so-called circular law [3, Thm. 1.10] says that the eigenvalue distribution (called the empirical spectral distribution) of $M_n/\sqrt{n}$ approaches the uniform distribution on the unit disk. This universality is quite remarkable: only the mean and variance matter. However, when the matrices are required to be unit upper Hessenberg with zero diagonal the circular law no longer appears to hold, and the eigenvalues may sometimes better be described as being distributed over a lozenge or other polygonal figure instead of a disk [1].

Little seems to be known yet about this effect of matrix structure. Looking closely at the edges of our plot we see a suggestion of an emerging fractal, which we have seen similar signs of in other upper Hessenberg Toeplitz Bohemian families. Finally, if we compute eigenvalues exhaustively (which we can only do for small dimensions) then we see evidence of what are being called "algebraic starscapes" [2], which are images of algebraic numbers and their Diophantine approximations.

For more details, see www.bohemianmatrices.com.

## FURTHER READING

[1] E.Y.S. Chan, R.M. Corless, L. Gonzalez-Vega, J.R. Sendra, J. Sendra and S.E. Thornton, Upper Hessenberg and Toeplitz Bohemians, Linear Algebra Appl. 601 (2020) 72–100.
[2] E. Harriss, K.E. Stange and S. Trettel, Algebraic Number Starscapes, arXiv:2008.07655.
[3] T. Tao, V. Vu and M. Krishnapur, Random Matrices: Universality of ESDs and the Circular Law, Ann. Probab. 38(5) (2010) 2023–2065.

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Rob is an Emeritus Distinguished University Professor at Western University, Canada. His research interests include computational dynamical systems, computational algebra, and computational special functions.

**Nicholas Higham**
Nick is a Royal Society Research Professor and Richardson Professor of Applied Mathematics at the University of Manchester. His research focuses on numerical linear algebra.

**Steven Thornton**
Steven is a recent PhD graduate of Western University, Canada. His research interests extend beyond Bohemian matrices into mathematical finance including algorithms for portfolio construction, and statistical arbitrage.