The Gatlinburg Symposia and Their Influence on the Development of Numerical Linear Algebra

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Abstract

About 1960, Alton S. Householder initiated the idea of "Gatlinburg Symposia on Numerical Linear Algebra". They were named after the resort of Gatlinburg, Tennessee, where the early meetings took place; later on they shifted to various other locations in North America and also in Europe. Responsible for their program was the "Gatlinburg Committee"; the first consisted of A.S. Householder, J.H. Wilkinson, W. Givens, G.E. Forsythe, P. Henrici, and F.L. Bauer. There were only invited lectures and there were, at least initially, no parallel sessions. The list of participants contains many well known names and continues to read like a "Who's Who" in the area of Numerical Linear Algebra, so that it is not surprising that the Symposia have had a tremendous influence on its development, both with respect to theory and the design of reliable and efficient algorithms. Such well-known software packages as LINPACK, LAPACK, EISPACK, and SPARSEPACK had their root in the collection of ALGOL programs in the book of Wilkinson and Reinsch, *Linear Algebra*, in many critical discussions during Gatlinburg Symposia, and in the close cooperation of scientists attending these meetings.

1 Background

In 1961, the proposal for a series of Gatlinburg Symposia was initiated by Alton S. Householder (1904-1993). He was then Director of the Mathematics Division of Oak Ridge National Laboratory and Ford Professor at the University of Tennessee.


Alston S. Householder

The background of his initiative was the new situation in numerical analysis, which had changed radically with the advent of digital computers and their growing availability at universities. Between 1950 and 1960 and before, only a few institutions associated with universities and government (like the National Bureau of Standards, the Oak Ridge National Laboratory and the Rand Corporation) had access to digital computers. In particular, many of the interested scientists concentrated at the National Bureau of Standards near Washington D.C. were eager to face the challenges of the new situation, and they knew the many results of classical mathematics that still could be used.

Using a digital computer was then tedious: programs had to be written in machine code or later in assembler language, as command driven languages like Fortran were still in their infancy. It was quickly seen that traditional numerical methods were inadequate when trying to realize them on digital computers: not only was it difficult to program them, but many of them turned out to be numerically unstable. A new foundation of numerical analysis and in particular of numerical linear algebra was necessary. This inspired the development of new methods and concepts in the years 1950-1960.

In 1950, D. Young proposed the SOR Algorithm, leading to new iterative methods for solving linear equations. This was systematically enhanced by the book of R. S. Varga, Matrix Iterative Analysis (1962). About the same time C. Lanczos (1950) and W. E. Arnoldi (1952) proposed Krylov-subspace methods. In 1952, M. Hestenes and E. Stiefel excited the community by their Conjugate Gradient Algorithm, because their method combined features of an iterative and a finite algorithm for solving linear equations.

In 1958, H. Rutishauser proposed the radically new LR Algorithm for computing the eigenvalues of a matrix, which was quickly extended by J.G. Francis (1961/62) to the now famous backward stable QR method. The basic idea, but without essential practical modifications, was also proposed by V.N. Kublanowskaya (1961) in the USSR.

The influence of round-off errors was analyzed by J. H. Wilkinson (1960, 1963) in a novel way, leading to the distinction between the condition number of a mathematical problem and the numerical stability properties of an algorithm to solve it. He developed so-called backward error analysis to judge the stability of an algorithm. The first formal backward error analysis was presented by W. Givens in 1954 in a technical report put out by the Oak Ridge National Laboratory.

In 1958 W. Givens also introduced the tool now called Givens rotations to numerical linear algebra; it was supplemented by the equally important tool introduced by Householder (1958), now called Householder matrices, which are reflectors, not rotators.

In 1947 G. B. Dantzig proposed the Simplex Method (see Dantzig (1963)) for solving a linear program (optimizing a system of linear inequalities subject to linear constraints), thereby starting the development of optimization methods which play a crucial role in many branches of engineering and other fields.

The ensuing rapid development of numerical analysis gave rise to new journals, such as Mathematics of Computation in 1954 (formerly Mathematical Tables and other Aids to Computation, 1943) and Numerische Mathematik in 1959 (see Figure 1 in the Appendix).

2 The Gatlinburg Symposia and Their Style

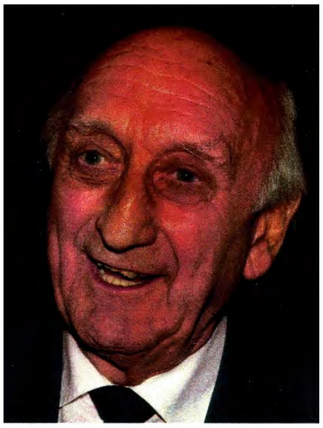
The Gatlinburg Symposia, officially started by Alston S. Householder in 1961, were influenced in part by the existence of two earlier and similar symposia, the first organized by Olga Taussky in 1951 in Los Angeles and the second by Wallace Givens in 1957 in Detroit. Householder had a special gift for persuading other top scientists to cooperate with him in launching a new conference series to foster a new foundation for numerical analysis, in particular for numerical linear algebra, made essential by the advent of digital computing. Due to his reputation and influence, Householder obtained the financial support of the National Science Foundation, allowing all participants of the early symposia to be fully paid.

The style of the Symposia was unusual. They were originally designed to be "closed" meetings (which, later on, led to growing concern and dislike by many about this particular aspect of the symposia). The reason for closed meetings was simply to limit the number of participants to about 100, so as to ensure that participants really could interact with one another.

Accordingly, the Symposia resembled more forums for discussion among scientists interested in computing, rather than traditional conferences to present results. There were no conference proceedings and no obligation for the participants to give a lecture. This loose structure was controlled by a Committee that assisted Householder to prepare and run the symposium. This Committee issued the invitations and, using a nice formulation of Richard Varga (1990), helped to "prompt" people to offer to give "spontaneous" lectures. There was always a full, but not overloaded, regular program with lectures of variable length and ample time for intensive discussions between them. Also, there were no parallel sessions in the early Symposia, and the official program was always supplemented by evening lectures organized on the spot.

The Committee of Gatlinburg III, shown on the left, is well known as this photo appears in the MATLAB Users Guide. The members, left to right, are James H. Wilkinson, Wallace Givens, George E. Forsythe, Alston S. Householder (Chair), Peter Henrici, and Friedrich L. Bauer.

The membership of the Committee could change with each successive meeting. For instance, the Gatlinburg IV Committee in 1969 consisted of F.L. Bauer, Miroslav Fiedler, A.S. Householder (Chair), A.M. Ostrowski, John Todd, Richard S. Varga, and J.H. Wilkinson. By 2005 the Householder XVI Committee was Angelika Bunse-Gerstner, Tony F. Chan, Alan Edelman, Nicholas J. Higham, Roy Mathias, Dianne P. O'Leary, Michael L. Overton, Henk A. van der Vorst, Paul Van Dooren, and Charles F. Van Loan (Chair).



F.L. Bauer

J.H. Wilkinson

The Committees were always careful to invite not only well known scientists but also promising young postgraduates, and to include all areas having both algebraic aspects and numeric content or implications. All this is illustrated by the program of Gatlinburg IV and the (partial) copy of its list of participants in Figures 3, 4, and 5 in the Appendix. In the early meetings an effort was made to interest experts in pure matrix theory but that did not succeed and an unfortunate separation emerged between these two related areas.

The attendance at the Symposia was international, in particular, there were always participants (already in the early Symposia) from behind the Iron Curtain. The author of this paper still remembers the sad discussion in April 1969 among the Czech attendants of Gatlinburg IV (I. Babuska, M. Fiedler, V. Pták and I. Marek), when they learned during the Symposium that the reform government of their country had been crushed by a Soviet invasion.

The first four Symposia took place in Gatlinburg, a resort village in Tennessee. Later on, the workshop moved to other places in North America and Europe. After Gatlinburg X, the first to return to Tennessee, they were renamed Householder Symposia on Numerical Linear Algebra (see Table 1 below).

Also Gatlinburg X was the first "open" meeting. Each person wishing to participate was requested to submit a short abstract of his or her research interest, and the Committee decided on whom to invite, based on this information.

Since 1970, a Householder prize for the best Ph.D. thesis has been awarded during each Symposium. Its subject should be (broadly) related to Numerical Linear Algebra. Applications for the prize, accompanied by an appraisal by the thesis advisor, are invited. The Committee decides on the recipient and announces the winner during the Symposium, who is expected give a lecture. The prize is entirely financed by voluntary contributions of the Conference participants. So far, the Householder Award has been granted to the following:

1971: F. Robert (Grenoble)
1974: Ole Hald (New York U.)
1977: Daniel D. Warner (UCSD)
1981: E. Marques de Sá (Coimbra) and Paul Van Dooren (K. U. Leuven)
1984: Ralph Byers (Cornell U.) and James M. Demmel (UC Berkeley)
1987: Nicholas J. Higham (U. of Manchester)
1990: Alan Edelman (MIT), Maria Beth Ong (U. of Washington)
1993: Hong-Guo Xu (Fudan U.) and Barry Smith (New York U.)
1996: Ming Gu (Yale U.)
1999: Jörg Liesen (U. Bielefeld)
2002: Jing-Rebecca Li (MIT)
2005: Jasper van den Eshof (U. Utrecht)
2008: David Bindel (UC Berkeley)

Table 1

Gatlinburg Symposia

No.	Year	Place (Organizers)
I	1961	Gatlinburg, USA (A.S. Householder)
II	1963	Gatlinburg, USA (A.S. Householder, F.W.J. Olver)
III	1964	Gatlinburg, USA (A.S. Householder)
IV	1969	Gatlinburg, USA (A.S. Householder)
V	1972	Los Alamos, USA (R.S. Varga)
VI	1974	Hopfen am See, Germany (F.L. Bauer)
VII	1977	Asilomar, USA (G.H. Golub)
VIII	1981	Oxford, England (L. Fox, J.H. Wilkinson)
IX	1984	Waterloo, Canada (J.A. George)
X	1987	Fairfield Glades, USA (R.C. Ward, G.W. Stewart)

Householder Symposia

No.	Year	Place (Organizers)
XI	1990	Tylosand, Sweden (Å. Björck)
XII	1993	Lake Arrowhead, USA (T.F. Chan, G.H. Golub)
XIII	1996	Pontresina, Switzerland (W. Gander, M.H. Gutknecht, D.P. O'Leary)
XIV	1999	Chateau Whistler, Canada (J.M. Varah, G.W. Stewart)
XV	2002	Peebles, Scotland (P. Knight, A. Ramage, A. Wathen, N.J. Higham)
XVI	2005	Campion, USA (J. Barlow, D. Szyld, H. Zha)
XVII	2008	Zeuthen, Germany (L. Liesen, V. Mehrmann, R. Nabben)

Householder VIII, Pontresina, Switzerland

3 Main Topics of the Symposia and Their Influence

The Symposia served as a forum for discussion among scientists about all aspects of using digital computers as a tool for the design and realization of reliable mathematical algorithms. This influenced and started developments not only in numerical mathematics but also in areas later belonging to computer science. For example, the design of the new well defined procedural programming language ALGOL was supported, in order to have a clean shareable tool for programming numerical software. This led to the so-called "handbook project" established in 1965 under the roof of Springer Verlag, in cooperation with Numerische Mathematik. It was essentially J.H. Wilkinson and his co-workers, mainly C. Reinsch, who established a fixed pattern to describe numerical software. Each program was accompanied by a description of the theoretical background of the underlying algorithm, of its applicability, and of its formal parameter list that precisely defined the meaning of its input and output parameters. Then followed a well-structured and syntactically correct ALGOL program, the description of its organisational and notational details, a discussion of numerical properties of the algorithm, typical examples of its use, and finally test results, illustrating the behaviour of the program in critical situations.

The handbook project started with prepublications in Numerische Mathematik in 1965, then collected in the famous *Handbook of Linear Algebra* ("HALA") of J.H. Wilkinson and C. Reinsch, which appeared in 1971 as Volume II of the *Handbook for Automatic Computation* published by Springer Verlag (see Figure 2 in the Appendix). Volume I, part a, on ALGOL itself, was written by H. Rutishauser in 1967. Although he had a profound influence on numerical linear algebra, poor health prevented him from attending more than one Gatlinburg meeting. Many attendants of the Symposia contributed to Volume II and to other related software packages that appeared later on.

As only a few others, Wilkinson (1965) strongly influenced the Gatlinburg Symposia and the contributions to "HALA" by his monograph *The Algebraic Eigenvalue Problem*. Using many classical results for his analysis, he described numerous new numerically stable algorithms for eigenvalue problems and improved many old methods.

Another new journal, Linear Algebra and Its Applications, was founded in 1968 with several attendants of the Symposia on its board of editors.

To the main subjects of the earlier Symposia belonged the QR algorithm and its properties (convergence, shift techniques, implicit shifts). The book of Wilkinson (1965) had stimulated the development of related algorithms. It led to the generalization of the QR method for computing the SVD (singular value decomposition) of matrices (Golub, Reinsch 1971).

Several results of classical mathematics also became of renewed interest in the context of numerical linear algebra, for instance, the moment problem, orthogonal polynomials, Gaussian inegration, summation methods, and approximation theory. Bauer and Householder (1960) estimated the eigenvalues of matrices in terms of moments. Other estimates were tied to classical results of Gerschgorin (see Varga (1962). A systematic study is found in Varga (2004). All these estimates turned out to be valuable tools for

G.H. Golub and R.S. Varga

G.W. (Pete) Stewart and W. Gautschi

investigating the sensitivity of the eigenvalue problem and the numerical stability of corresponding algorithms.
Orthogonal polynomials were used by Hestenes and Stiefel (1952) to estimate the approximation error of the iterates of their cg-algorithm (conjugate gradient). The classical role of these polynomials in weighted Gaussian integration was greatly enhanced by new results. The construction of such quadrature rules was considered by Gautschi (1968) and Golub and Welsch (1969), and is tied to the construction of positive definite tridiagonal matrices and the computation of their eigenvectors. The qd-algorithm of Rutishauser (1954) is closely related to his LR-algorithm, but also gave rise to new summation methods to speed up the convergence of sequences and series. In particular, this created new interest in Richardson-extrapolation techniques and their application to ordinary and partial differential equations.

To the highlights of Gatlinburg V (1972) in Los Alamos belong the presentation by Pete Stewart of the QZ algorithm for solving $Ax = \lambda Bx$ (C. Moler, G.W. Stewart (1971)), evoking the admiration (coupled with admitted envy) of Jim Wilkinson. This QZ paper launched the algorithmic study of generalized eigenvalue problems and matrix pencils, and the development of a perturbation theory for them (Stewart (1978), Sun (1977)).

The ALGOL program collection in HALA stimulated enlarged software packages of FORTRAN programs written in the style of HALA, EISPACK (Smith et al. 1976, 1977) and LINPACK (Dongarra et al. 1979).

A major theme since Gatlinburg VII has been the exploitation of sparsity in direct methods for solving linear equations. Preserving sparsity and numerical stability by proper pivoting led to the program package SPARSEPACK (George, Liu, Ng (1980)) and to the books of George, Liu (1981), and Duff, Erisman and Reid (1986). To enhance the use of vector machines in numerical linear algebra, the development of BLAS (basic linear algebra subroutines) was started, and they were incorporated into the software package LAPACK (Anderson et al. (1992)).

For distributed memory computers, a suitable program package is ScaLAPACK (L.S. Blackford et al. (1997)), which is based on LAPACK. The importance of all these packages is also seen from the fact that they form an essential part of current test-beds to assess the speed of today's supercomputers and the quality of their operating systems. Many of these routines were incorporated into MATLAB, MATHEMATICA and MAPLE. At several Symposia, during the coffee breaks between lectures, Cleve Moler demonstrated the easy use of MATLAB to solve problems in numerical linear algebra, and impressed an ever growing audience.

To the regular visitors of the Symposia belonged W. (Velvel) Kahan. His critical remarks were feared by all, but helped to improve many results; in particular his repeated and insistent critique of the low quality of the arithmetic operations of the then available computers finally led to the IEEE standard for computer arithmetic, adopted in 1985 and still in current use.

Major themes of the Symposia originated from a fresh view of the cg algorithm as an iterative method, thanks to J. Reid (1971), and then to Krylov spaces. This led to investigations of a whole nest of problems:

– the relation of the cg-algorithm to the algorithms of Lanczos (1950) and Arnoldi (1951)
– the solution of the symmetric eigenvalue problem (Parlett, 1980)
– the Kaniel-Paige theory (Kaniel 1966, Paige 1971)
– solving linear equations with a symmetric but indefinite matrix (Paige, Saunders (1975))
– the GMRES method of Saad and Schultz (1986)
– the fast but unreliable BI-CG method (Lanczos (1950), Fletcher (1976)) and stabilizations of it
– the Bi-CGSTAB-method (van der Vorst 1992) presented at Gatlinburg XI
– the QMR-method (Freund, Nachtigal (1991)), also presented at Gatlinburg XI

The convergence of all these iterative methods can be sped up by preconditioning techniques, a recurrent topic of the Symposia. With very large problems coming from partial differentiation in important applications, in order to be able to solve such problems at all, it became necessary to adapt algorithms for finite element methods, multigrid algorithms and domain decomposition in order to take into account the special structure of matrices, and this became one of the regular topics of the Symposia.

Filtering and control were increasingly addressed at the Symposia. The use of the the Kronecker normal form in relevant algorithms was studied by P. van Dooren (1979), who won the Householder prize of Gatlinburg VIII in 1981. The late R. Byers (1983) won the Householder prize of Gatlinburg IX in 1984 for the solution of the algebraic Riccati equation by exploiting the structure of Hamiltonian and symplectic matrices. Statistical calculations profit very much from the increasingly refined arsenal of SVD related algorithms.

Early contributions of numerical linear algebra to the broad area of optimization should also be noted. This was to be expected as the solution of positive definite linear systems is equivalent to the minimization of a strictly convex quadratic function. It motivated new algorithms for solving linearly constrained quadratic programs and, more generally, to solve nonlinearly constrained nonlinear programs (by so-called sequential quadratic programming (SQP) methods). Even though these methods for nonlinear problems are the proper subject of other societies like the Mathematical Programming Society, their realization led to interesting specially structured problems of numerical linear algebra also considered during the Gatlinburg/Householder Symposia. An example is the MINOS program package started by Murtagh and Saunders in 1977 (see Murtagh and Saunders (1978)), with later additions by P.E. Gill, W. Murrray and M.H. Wright. There are relations to the interior point methods of mathematical programming since all these methods have to repeatedly solve large sets of linear equations with a similar structure.

These remarks also apply to other modern optimization problems such as semidefinite programming. Here one considers linear programs in the Hilbert space (with respect to the Frobenius norm) of symmetric matrices, ordered by the cone of positive definite matrices. These programs lead to intricate problems of numerical linear algebra that have been addressed during the Symposia, e.g. by M. Overton during the Symposium of 1996. Also see the later paper by Alizadeh, Haeberly and Overton (1998).

All this shows that there are fruitful relations between numerical linear algebra and various areas of applications. These areas typically lead to consideration of new problems with an interesting mathematical structure, which require efficient and stable algorithms of numerical linear algebra to solve them. This interplay is fostered by the Gatlinburg/Householder Symposia. From their beginning, they have served as meetings of mathematicians who are willing to face the problems connected with the development of such methods.

Acknowledgement. The author thanks Beresford Parlett for the careful reading and correction of a draft of this paper, and Walter Gander and Martin Gutknecht for providing photos from the Householder Conference XIII in Pontresina. Also the author gratefully admits that he had access to the reminiscences of Richard Varga (1990) on the Gatlinburg Symposia.

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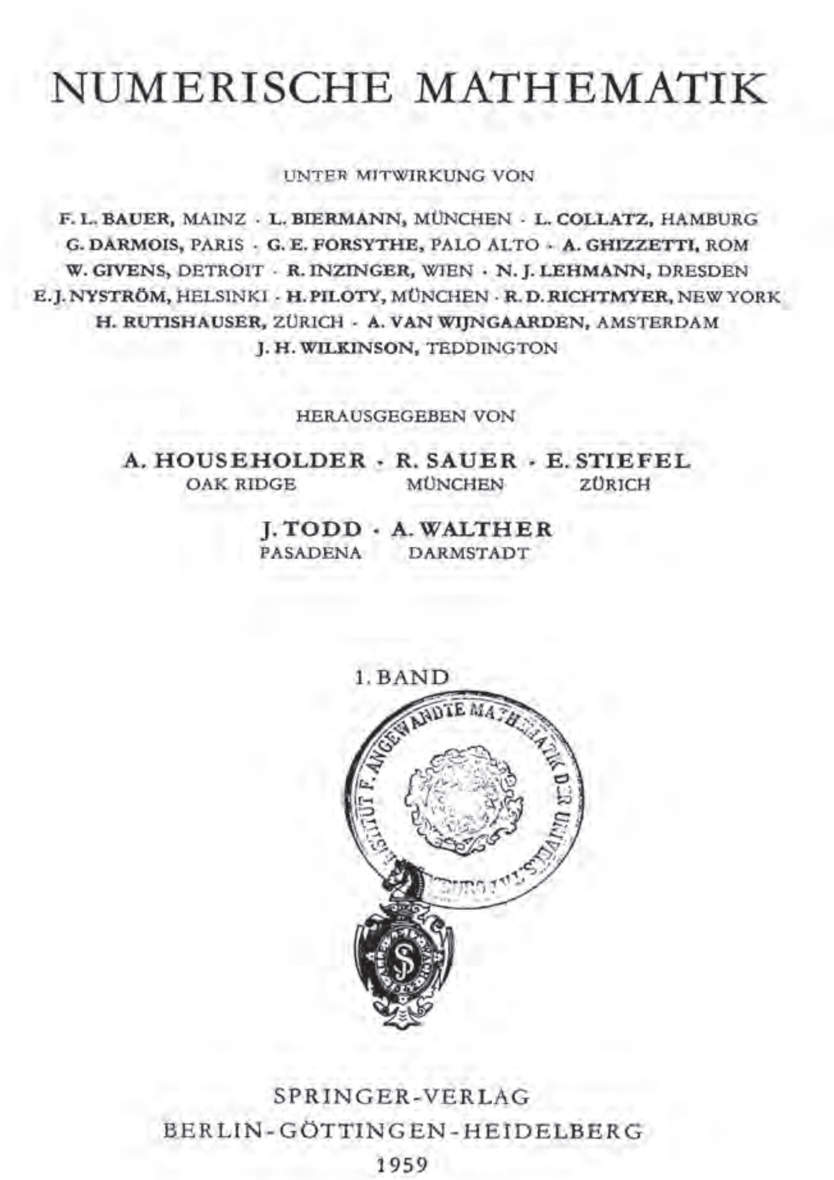
Ed. Note: A brief history of the Householder Symposia is at http://www3.math.tu-berlin.de/householder_2008/history.php. An interested reader might also look at http://www.mathworks.com/company/newsletters/news_notes/clevescorner/dec04.html (where Cleve Moler reminisces about the Gatlinburg meetings and shows more photos of the early organizers), as well as articles about numerical linear algebra that have appeared in IMAGE: "The ILAS 2008 Hans Schneider Prize in Linear Algebra Goes to Beresford Parlett and Cleve Moler", issue 40; "Finding John Francis Who Found QR Fifty Years Ago", issue 43; and "Hans Schneider Award Winners Celebrated", issue 45. All issues of IMAGE may be viewed at http://www.ilasic.math.uregina.

A Appendix

Figure 1: Numerische Mathematik



NUMERISCHE MATHEMATIK

UNTER MITWIRKUNG VON

F. L. BAUER, MAINZ · L. BIERMANN, MÜNCHEN · L. COLLATZ, HAMBURG
G. DARMOIS, PARIS · G. E. FORSYTHE, PALO ALTO · A. GHIZZETTI, ROM
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PASADENA · DARMSTADT

1. BAND

SPRINGER-VERLAG
BERLIN-GÖTTINGEN-HEIDELBERG
1959

Figure 2: Handbook

Handbook for Automatic Computation

Edited by

F. L. Bauer · A. S. Householder · F. W. J. Olver

H. Rutishauser † · K. Samelson · E. Stiefel

Volume II

J. H. Wilkinson · C. Reinsch

Linear Algebra

Chief editor

F. L. Bauer

Springer-Verlag Berlin Heidelberg New York 1971

Figure 3: Program of Gatlinburg IV

PROGRAM

FOURTH GATLINBURG SYMPOSIUM ON NUMERICAL ALGEBRA

Mountain View Hotel April 14-19, 1969

Monday, April 14, 1969

9:30	Welcome	A. S. Householder
9:45	Some Basic Results in Automatic Solution of Polynomial Equations	A. M. Ostrowski
11:15	Convergence Theorems for Secant Methods in n Dimensions	J. M. Ortega
2:00	Alternating Direction Methods for Galerkin Approximations for Parabolic Problems	J. Douglas
3:30	The Computational Aspects of Applying Variational Techniques to Boundary Value Problems	R. S. Varga

Tuesday, April 15, 1969

9:30	On the Numerical Improvement of Optimizing Algorithms	J. Stoer
11:00	Another Algorithm for Minimizing a Sum of Squares of Nonlinear Functions	M. J. D. Powell
2:00	Bounds for the Second Eigenvalue	F. L. Bauer
3:30	Techniques for the Eigenvalue Problems	N. A. Gastinel

Wednesday, April 16, 1969

9:30	Natural Norms in Algebraic Processes	V. N. Faddeeva
11:00	Generalized Eigenvalue Problem	V. N. Kublanovskaja

- Afternoon free -

Thursday, April 17, 1969

9:30	On Some Classes of Matrices	M. Fiedler
11:00	The Critical Exponent and the Spectral Radius of Matrices	V. Pták
2:00	The $Ax = \lambda Bx$ and Related Problems	J. H. Wilkinson
3:30	Some New Results on the QR and LR Algorithms	J. F. Traub

Friday, April 18, 1969

9:30	Matrix Bound Norms	H. Schneider
11:00	Bounds on the Eigenvalues of a Matrix	A. J. Hoffman
2:00	Bounds for the Abscissa of Stability of a Stable Polynomial	P. Henrici
3:30	The Method of Odd/Even Reduction and Factorization with Application to Poisson's Equation	G. H. Golub

Saturday, April 19, 1969

9:00	Sectionally Linear Functions	H. Schwerdtfeger
10:30	On Some Finite Element Procedures	M. Zlámal

Figure 4: Participants of Gatlinburg IV

PARTICIPANTS

FOURTH GATLINBURG SYMPOSIUM ON NUMERICAL ALGEBRA

Mountain View Hotel April 14-19, 1969

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Figure 5: Participants of Gatlinburg IV, ctd.

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