MATH36001 Matrix Analysis

Credit Rating	10
Level	Third Level
Delivery	Semester One
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Aims

To introduce students to matrix analysis through the development of essential tools such as the Jordan canonical form, Perron-Frobenius theory, the singular value decomposition, and matrix functions.

Brief description

This module is an introduction to matrix analysis, developing essential tools such as the Jordan canonical form, Perron-Frobenius theory, the singular value decomposition, and matrix functions. It builds on the first year linear algebra course. Apart from being used in many areas by almost all mathematicians, Matrix Analysis has broad applications in fields such as engineering, physics, statistics, econometrics and data mining, and examples from some of these areas will be used to illustrate and motivate some of the theorems developed in the course.

Intended learning outcomes

On completion of this unit successful students will

ILO 1 Construct some key matrix decompositions such as the Jordan canonical form of a square matrix and the singular value decomposition of a rectangular matrix.

ILO 2 Localize the eigenvalues of a matrix using matrix norms and Gershgorin's theorem.

ILO 3 Check the solvability of a linear system and when solvable, provide the set of solutions in terms of the pseudoinverse or the singular value decomposition of the matrix of the linear system.

ILO 4 Solve least squares problems and construct low-rank approximations to a matrix.

ILO 5 Compute the matrix exponential and use it for solving differential and algebraic equations.

ILO 6 Apply Perron's theorem to positive matrices and the Perron-Frobenius theorem to nonnegative matrices.

Prerequisites

MATH10202, MATH10212 (Linear Algebra).

Syllabus

1. **Basics.** Summary/recap of basic concepts from linear algebra including matrices and vectors, determinants, singularity of matrices, rank. , eigenvalues and eigenvectors.

2. **Canonical forms.** Reduction of square matrices to simpler form including spectral decomposition of Hermitian matrices, Schur decomposition and Jordan canonical form; minimal and characteristic polynomials, Cayley-Hamilton Theorem.

3. **Norms.** Vector norms and matrix norms, bounds for eigenvalues, Gershgorin theorem.

3. **Generalized inverses and singular value decomposition (SVD).** Projectors; pseudo-inverses; application to linear least squares; polar decomposition.

4. **Nonnegative matrices and related results.** Irreducible matrices; Perron's theorem and Perron-Frobenius theorem;

5. **Matrix exponential.** Definitions of the matrix exponential function and application to the solution of differential equations and higher order equations.

Textbooks

Teaching and Learning Methods

Two lectures per week, with a weekly examples class.

Learning Hours

Activity	Hours
Staff/student contact	36
Private study	60
Total	96

Assessment

An end of module 2-hour examination (80%) and mid-term test (20%).

Core Learning Materials