MATH31001/41001/61001: LINEAR ANALYSIS

Example Sheet 2

1. Using the Weierstrass Approximation Theorem, prove that there exists a countable dense subset of $C[0,1]$. *(Hint: the polynomials with real coefficients are dense but not countable; how can one modify this example?)*

2. Verify that the sets ℓ^1 and ℓ^∞ are indeed vector spaces.

3. Let $c_0 = \{(a_i)_{i=0}^\infty : \sup_i |a_i| < +\infty \text{ and } a_i \to 0 \text{ as } i \to \infty\}$. Let $\|(a_i)_{i=0}^\infty\|_\infty = \sup_i |a_i|$. Show that c_0 is a closed subspace of l^∞.

4. Is the set of monotonically increasing functions on $[0,1]$ (i.e., $f : f(x) \leq f(y)$ if $x \leq y$) a vector space?

5. Is the set of monotonic functions on $[0,1]$ (i.e., f such that f is either increasing or decreasing) a vector space?

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