Automorphisms of Homogenous Finitary Symmetric Groups

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Dedicated to the memory of Brian Hartley

Let $\kappa$ be an arbitrary infinite cardinal number. Let $F\text{Sym}(\kappa)$ denote the finitary symmetric group and $Alt(\kappa)$ denote the alternating group on the set $\kappa$. Let $\Pi$ be a set of sequences of prime numbers and $\xi \in \Pi$. Then $\xi$ is a sequence of not necessarily distinct primes. Let $\alpha \in F\text{Sym}(\kappa)$, ($Alt(\kappa)$). For a natural number $p \in \mathbb{N}$, a permutation $d^p(\alpha) \in F\text{Sym}(\kappa p)$ defined by $(\kappa s + i)^{d^p(\alpha)} = \kappa s + i^p$, $i \in \kappa$ and $0 \leq s \leq p - 1$, is called a homogeneous $p$-spreading of the permutation $\alpha$. We divide the ordinal $\kappa p$ into $p$ equal parts and on each part we repeat the permutation diagonally as in the finite case. We continue to take the embeddings using homogeneous $p$-spreadings with respect to the given sequence of primes in $\xi$. From the given sequence of embeddings, we have direct systems and hence direct limit groups $F\text{Sym}(\kappa)(\xi)$, ($Alt(\kappa)(\xi)$). Observe that $F\text{Sym}(\kappa)(\xi)$ and $Alt(\kappa)(\xi)$ are subgroups of $\text{Sym}(\kappa \omega)$ for details see [1].

We now answer the question that when are two strictly diagonal type groups isomorphic?

**Theorem 1. (Kegel-Kuzucuoğlu)** Let $\kappa$ be an infinite cardinal. If $G = \bigcup_{i=1}^{\infty} G_i$, where $G_i = F\text{Sym}(\kappa n_i)$, ($H = \bigcup_{i=1}^{\infty} H_i$, where $H_i = Alt(\kappa n_i)$), is a group of strictly diagonal type and $\xi = (p_1, p_2, \ldots)$, then $G$ is isomorphic to the homogenous finitary symmetric group $F\text{Sym}(\kappa)(\xi)$ ($H$ is isomorphic to homogenous alternating group $Alt(\kappa)(\xi)$), where $n_0 = 1$, $n_i = p_1 p_2 \ldots p_i$, $i \in \mathbb{N}$.

By Theorem 1, infinite homogenous finitary symmetric groups are isomorphic if and only if their orders and characteristics are equal. Hence the isomorphism classes of strictly diagonal type can be parameterized by the cardinality of the group and the corresponding Steinitz number and we obtain the complete classification up to isomorphism of such groups.

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Corollary 2. (Kegel-Kuzucuoğlu) If the characteristics of two homogenous finitary symmetric groups are different (homogenous infinite alternating groups), then the groups are non-isomorphic.

The automorphisms of the groups $S(\xi)$ are studied by Lavrenyuk and Sushchanskyin [3]. If time permits we will discuss the structure of some of the automorphism of the groups $FSym(\kappa)(\xi)$.

References


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