Make sure that you include, at least, some comments in your code. Try to use long and descriptive names for variables in your code. Worked solutions to the problems can be found on my website (click here).

Random Numbers

See the reference website http://en.cppreference.com for details on random number generation (click here).

3.1 Declare the type `mt19937 rng;` in your code. This will declare a random number generator called rng.

3.2 The function `rng()` will return a sequence of random integer values. Output the first 100 numbers in the sequence.

3.3 Use the type `uniform_real_distribution<double> U(0.,1.);` to create a uniform distribution between 0 and 1. Call the next number in the sequence with the command `U(rng)` Calculate the probability that a random draw \( u \) is between 0.25 and 0.5.

3.4 Do your numbers appear random if you run the code multiple times? Use the function `rng.seed(int)` to reset your random number sequence.

3.5 The type `random_device randomSeed;` can be used to generate a hardware dependent random start point for the sequence. Send a random seed to the sequence with the command `rng.seed(randomSeed());` . Note that this function only need ever be called once at the beginning of your program.

3.6 Use the type `normal_distribution<double> Phi(0.,1.);` to create a normal distribution with mean 0 and variance 1. Generate a histogram to count the number of times the result lands in the \( i \)th interval where

\[
a = -4.2 \leq (i - 1/2)h < x < (i + 1/2)h \leq b = 4.2
\]

with \( i = -10, -9, \ldots, 9, 10 \) and \( h = 0.4 \).
**Monte-Carlo Integration**

Here we must use normally generated random numbers to calculate the value of a Black Scholes call option \( C \) with strike price \( X = 100 \) and a maturity \( T = 1 \) that is one year from now, given the current stock price \( S_0 = 104.81 \). Assume we can price under the risk neutral measure, and that \( S \) follows a GBM with \( \sigma = 0.4 \), the interest rate is \( r = 0.03 \).

3.7 Code up the problem to return an estimate for the option value.

3.8 Once the algorithm has been developed inside main, move it out into its own function. Think about the inputs and outputs from the function.

3.9 Generate value for different numbers of samples, and output to file. Can you use excel to find the convergence rate of the algorithm and estimate the error if we use \( N = 1000000 \) samples?