MATH60082
Examples Sheet 2
Bonds and Euler Method

Bonds

2.1 You must price a coupon bond with a principle of \$100 maturing in 2 years time. Two coupons of \$5 are paid at the end of year one and year two, and the continuous compound interest rate is a constant 10%.

(i) Verify that the value of the bond is given by

$$V(t=0) = 5e^{-0.1\times 1} + 5e^{-0.1\times 2} + 100e^{-0.1\times 2}$$

Now write a program to return the result, using the variables *maturity*, *interestRate*, *coupon*, and *principle* to reference the appropriate parameters. Once your program is working you should always store a backup version of this file.

(ii) Next introduce the variables *couponTime* and *numberCoupons* to your program. The variable *couponTime* should denote the time in years between coupons, and *numberCoupons* should calculate the resulting number of coupons given a *maturity*. Now use a *for* loop to calculate the value of the same bond.

(iii) Try different values for your parameters. What happens if *maturity* is not divisible by *couponTime*? How can you change your program to fix any problems this might raise.

(iv) It is now easy to create a function to price bonds. Use the following structure to return the value:-

```
double bondPricer(double maturity,double interestRate,
                  double coupon,double principle,
                  double couponTime)
{
  // fill this in
}
```

Solving ODEs

2.2 The Euler method for integrating an ODE

$$\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0$$

is given by the algorithm

$$y_{n+1} = y_n + hf(x_n, y_n), \quad n = 0, 1, \ldots$$

where $y_{n+1} = y(x_n + h)$ and h is the step size.

(i) Write a program to solve the following dierential equation

$$\frac{dy}{dx} = xe^{3x} - 2y, \quad 0 \leq x \leq 1, \quad y(0) = 0.$$

Use step sizes $h = .1, .01, .001$ and tabulate your results.

(ii) Plot a figure to compare your results with the exact solution to the problem given by

$$y(x) = \frac{1}{5}xe^{3x} - \frac{1}{25}e^{3x} + \frac{1}{25}e^{-2x}.$$

(iii) The Modified Euler method is given by

$$y_{n+1} = y_n + hf(x_n + h/2, y_n + (h/2)f(x_n, y_n)), \quad n = 0, 1, \ldots$$

i. Resolve the problem using this method and compare your solutions to the Euler method and exact solution.

ii. Which method is best?

iii. Can you find the rate of convergence?

References