1. We have

\[ C(S, t) = e^{-D_0(T-t)}C_1(S, t), \]

where \( C_1(S, t) \) satisfies the classical Black-Scholes equation with \( r \) replaced by \( r - D_0 \). The value of \( C_1(S, t) \) is therefore just that of a normal European call with interest rate \( r - D_0 \):

\[ C_1(S, t) = SN(d_{10}) - Ee^{-(r-D_0)(T-t)}N(d_{20}), \]

where

\[ d_{10} = \frac{\ln(S/E) + (r - D_0 + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}, \]

\[ d_{20} = d_{10} - \sigma\sqrt{T-t}. \]

2. In this case \( D_0 = 0.01 \), \( S_0 = 80 \), \( E = 60 \), \( T = 0.25 \), \( \sigma = 0.3 \) and \( r = 0.1 \). First, we compute the values of \( d_{10} \) and \( d_{20} \) at \( t = 0 \):

\[ d_{10} = \frac{\ln(80/60) + (0.1 - 0.01 + (0.3)^2 \times 0.5) \times 0.25}{0.3 \times \sqrt{0.25}} \approx 2.1429 \]

\[ d_{20} = 2.1429 - 0.3 \times \sqrt{0.25} \approx 1.9929 \]

The value of call option is

\[ C_0 = Se^{-D_0T}N(d_{10}) - Ee^{-rT}N(d_{20}). \]

Since

\[ N(2.1429) \approx 0.9839, \quad N(1.9929) \approx 0.9769 \]

then

\[ C_0 = 80 \times e^{-0.01 \times 0.25} \times 0.9839 - 60 \times e^{-0.1 \times 0.25} \times 0.9769 \approx 21.349 \]

3. This exercise is very similar to one from problem sheet 6, part 4a.

\[ \Delta = \frac{\partial C}{\partial S} = e^{-D_0(T-t)}N(d_{10}). \]

4. If \( S \to \infty \), then \( d_{10} \to \infty \) and \( d_{20} \to \infty \). Therefore \( N(d_{10}) \to 1 \) and \( N(d_{20}) \to 1 \).

As \( S \to \infty \), it follows from

\[ C(S, t) = Se^{-D_0(T-t)}N(d_{10}) - Ee^{-r(T-t)}N(d_{20}) \]

that

\[ C(S, t) \to Se^{-D_0(T-t)}. \]

For large \( S \) it is certainly below the payoff \( C(S, T) = S - E \to S \), since \( e^{-D_0(T-t)} \) is less than one.