Lecture 4: Derivatives

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Lecture 4

1. Financial Derivatives
2. European Call and Put Options
3. Payoff Diagrams, Short Selling and Profit
A Derivative is a financial instrument whose value depends on the values of other underlying variables. Other names are financial derivative, derivative security, derivative product. A stock option, for example, is a derivative whose value is dependent on a stock price.

Examples: forward contracts, futures, options, swaps, CDS, etc.

Options are very attractive to investors, both for speculation and for hedging.

So what is an Option?
**Definition**

**European call option** gives the holder the right (not obligation) to buy underlying asset at a prescribed time $T$ for a specified (strike) price $E$.

**European put option** gives its holder the right (not obligation) to sell underlying asset at a prescribed time $T$ for a specified (strike) price $E$.

The question is:

“What does this actually mean?”
Example: European Call Options

Consider a three-month European call option on a BP share with a strike price $E = 15$ ($T = 0.25$). If you enter into this contract you have the right but not the obligation to buy one share for $E = 15$ in a three months time.

Whether you exercise your right depends on the stock price in the market at time $T$:

- If the stock price is above £15, say £25, you can buy the share for £15, and sell it immediately for £25, making a profit of £10.
- If the stock price is below £15, there is no financial sense to buy it. The option is worthless.
We denote by $C(S, t)$ the value of European call option and $P(S, t)$ the value of European put option.

**Definition**

Payoff Diagram is a graph of the value of the option position at expiration $t = T$ as a function of the underlying stock price $S$.

Call price at $t = T$:

$$C(S, T) = \max (S - E, 0)$$

$$= \begin{cases} 
0, & S \leq E, \\
S - E, & S > E,
\end{cases}$$
Put price at $t = T$:

$$P(S, T) = \max (E - S, 0)$$

$$= \begin{cases} 
  E - S, & S \leq E, \\
  0, & S > E, 
\end{cases}$$

If a trader thinks that the stock price is on the rise, he can make money by purchasing a call option without buying the stock. If a trader believes the stock price is on the decline, he can make money by buying put options.
The profit (gain) of a call option holder (buyer) at time $T$ is

$$\max (S - E, 0) - C_0 e^{rT},$$

where $C_0$ is the initial call option price at $t = 0$.

Example:
Find the stock price on the exercise date in three months, for a European call option with strike price £10 to give a gain (profit) of £14 if the option is bought for £2.25, financed by a loan with continuously compounded interest rate of 5%.

Solution:
$$14 = S(T) - 10 - 2.25 \times e^{0.05 \times \frac{1}{4}},$$

$$S(T) = 26.28$$

For the holder of European put option, the profit at time $T$ is

$$\max (E - S, 0) - P_0 e^{rT}$$
**Definition**

Short selling is the practice of selling assets that have been borrowed from a broker with the intention of buying the same assets back at a later date to return to the broker.

This technique is used by investors who try to profit from the falling price of a stock.

**Definition**

Portfolio is a combination of assets, options and bonds.

We denote by $\Pi$ the value of a portfolio. Example: $\Pi = 2S + 4C - 5P$.

It means that the portfolio consists of long position in two shares, long position in four call options and a short position in five put options.
Option positions

\[ C(S, T) \text{ Long Call} \]

\[ P(S, T) \text{ Long Put} \]

\[ -C(S, T) \text{ Short Call} \]

\[ -P(S, T) \text{ Short Put} \]
**Straddle** is the purchase of a call and a put on the same underlying security with the same maturity time $T$ and strike price $E$.

The value of portfolio is $\Pi = C + P$

- Straddle is effective when an investor is confident that a stock price will change dramatically, but is uncertain of the direction of price move.
Example of large profits: $S_0 = 40$, $E = 40$, $C_0 = 2$, $P_0 = 2$.
Can you find the expected return if the stock price at $T$ is given by the following tree?

$$S_0 = 40 \xrightarrow{p = \frac{3}{4}} 60 \xleftarrow{p = \frac{1}{4}} 20$$

Ans: 400%