Lecture 19: Asian Options

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Introduction to Financial Mathematics, 2015
1 Asian Options

2 Derivation of a PDE for Asian Options
An **Asian Option** is a contract giving the holder the right to buy/sell an underlying asset for its **average price** over some prescribed period.

The floating strike Asian put option has the final condition:

\[
V(S, T) = \max \left( S - \frac{1}{T} \int_0^T S(t) dt, 0 \right).
\]

We introduce a new variable:

\[
I(t) = \int_0^t S(t) dt \quad \text{or} \quad \frac{dI}{dt} = S(t).
\]

The final condition can now be written as

\[
V(S, I, T) = \max \left( S - \frac{I}{T}, 0 \right).
\]
Using Itô’s lemma:

\[ dV = \left( \frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} \right) dt + \frac{\partial V}{\partial S} dS + \frac{\partial V}{\partial I} dI \]

and setting up a hedging portfolio we find

\[ d\Pi = \left( \frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} \right) dt + \frac{\partial V}{\partial S} dS - \Delta dS + \frac{\partial V}{\partial I} dI \]

We can eliminate the random component in \( d\Pi \) by choosing \( \Delta = \frac{\partial V}{\partial S} \).
By using No-Arbitrage Principle, and the equation

\[ dI = Sdt \]

we can obtain the modified Black-Scholes PDE for the Asian option price:

\[
\frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV + S \frac{\partial V}{\partial I} = 0
\]

The value of an Asian option must be calculated numerically (no analytic solution!).