1. The stock price obeys the stochastic differential equation \( dS = \mu S dt + \sigma S dW \).

   (a) (2 marks) Suppose that the expected return from a stock is 16% per annum and the volatility is 45%. Initial stock price is $80. By using \( dW \approx X(\Delta t)^{1/2} \), where \( X \sim N(0,1) \), calculate the change \( \Delta S \) in the stock price during seven days.

   Solution:

   (b) (3 marks) By using Ito’s Lemma 
   
   \[
   df = \left( \frac{\partial f}{\partial t} + \mu S \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} \right) dt + \sigma S \frac{\partial f}{\partial S} dW,
   \]

   find the SDE satisfied by

   \( f(t,S) = AtS^2 \), where \( A \) is an arbitrary constant.

   Solution:

2. (a) (2 marks) Find a lower bound for the European call option with exercise price $15, when the current stock price is $50, the time to maturity is six months, and the risk-free rate of interest is 1% p.a.

   Solution:
(b) (3 marks) For the previous case, consider the situation, where the European call option is £5 which is less than the theoretical minimum. Show that there exists an arbitrage opportunity. (hint: you must create a portfolio with the option, shares and bonds)

Solution:

3. (4 marks) Draw the payoff diagram of the portfolio: long one share, short one call and short two puts, all with strike price E.

Solution:

4. (6 marks) Draw the payoff diagram of the portfolio: short two shares, long three puts with strike price $E_1$ and long four calls with strike price $E_2$ ($E_1 < E_2$).

Solution: