Recall that the Black-Scholes equation 
\[ \frac{\partial C}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS \frac{\partial C}{\partial S} - rC = 0 \]
has the explicit solution for the European call option:
\[ C(S,t) = SN(d_1) - Ke^{-r(T-t)}N(d_2), \]
where
\[ N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{y^2}{2}} dy, \quad d_1 = \frac{\ln(S/E) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}, \quad d_2 = d_1 - \sigma\sqrt{T-t}. \]

1. Show by substitution that the following functions are exact solutions of the Black-Scholes equation
   (a) \( C(S,t) = AS, \) (A is the arbitrary constant);
   (b) \( C(S,t) = S - Ke^{-r(T-t)} \) (the value of the forward contract, where \( K = \text{const} \) is the delivery price)

2. Find all parameters \( \alpha \) for which the function \( C(S,t) = S^\alpha e^{-r(T-t)} \) is the solution of the Black-Scholes equation.

3. Calculate the price of a three-month European call option on a stock with a strike price $60 when the current stock price is $80. The risk-free interest rate is 10% per annum. The volatility is 30%.

4. Using the analytic formula for the Black-Scholes equation, find the value of the call option in the limit \( \sigma \to 0. \)
Short Answers

1. (a) -
   (b) -

2. \( \alpha_1 = 0 \) and \( \alpha_2 = 1 - \frac{2r}{\sigma^2} \).

3. \( C_0 = 21.549 \)

4. 
   \[
   \lim_{\sigma \to 0} C(S, t) = \begin{cases} 
   0 & \text{if } S \leq E e^{-r(T-t)} \\
   S - E e^{-r(T-t)} & \text{if } S > E e^{-r(T-t)}
   \end{cases}
   \]