1. (Warming up). What is the interest rate if a deposit subject to continuous compounding is doubled after 12 years?

2. (Warming up). What is the difference between £140 deposited at 10% compounded monthly \( (m = 12) \) and compounded continuously after one year?

In what follows the stock price \( S(t) \) obeys the stochastic differential equation

\[ dS = \mu S dt + \sigma S dW, \]

where \( \mu \) is the expected return, \( \sigma \) is the volatility, \( W(t) \) is a standard Wiener process (Brownian motion).

3. Suppose that the expected return from a stock is 14% per annum and the volatility is 20%. Initial stock price is $90. By using \( dW \approx X \sqrt{\Delta t} \), where \( X \) is \( N(0,1) \), calculate the increase \( \Delta S \) in the stock price during three days.

4. Suppose that a stock price has an expected return of 36% per annum and a volatility of 40%. When the stock price at the end of a certain day is $80, calculate the following
   (a) the expected stock price at the end of the next day;
   (b) the standard deviation of the stock price at the end of the next day;

5. If \( dS = \mu S dt + \sigma S dW \), and \( A \) and \( n \) are constants, find the SDE satisfied by (use Ito’s Lemma)
   (a) \( f(S) = AS + t^2 \),
   (b) \( f(S) = S^{1/2} \),
   (c) \( f(S) = S^n \)

6. Suppose that initial stock price is $90, the expected return from a stock is 24% per annum, and the volatility is 35%. Find the distribution of \( \ln(S) \) in eight months’ time.

7. (not easy!!) Show that the probability density function \( p(y,t) \) for the Wiener process \( W(t) \)

\[ p(y,t) = \frac{\partial}{\partial y} P(W(t) \leq y) = \frac{1}{\sqrt{2\pi t}} \exp\left(-\frac{y^2}{2t}\right) \]

obeys the following differential equation

\[ \frac{\partial p}{\partial t} = \frac{1}{2} \frac{\partial^2 p}{\partial y^2}. \]
Short Answers

1. 5.78%.

2. about £0.06

3. \( \Delta S \approx 0.104 + 1.632X \)

4. (a) 80.079
   (b) 1.675

5. (a) \( df = (2t + \mu (f - t^2)) dt + \sigma (f - t^2) dW \)
   (b) \( df = (\mu / 2 - \sigma^2 / 8) f dt + \sigma / 2 f dW \)
   (c) \( df = (\mu n + \frac{1}{2} \sigma^2 n (n - 1)) f dt + \sigma n f dW \)

6. \( \ln S \sim N (4.619, 0.082) \)

7. -