For any primitive $(n+1)$–dimensional weight vector χ of positive integers, the weighted projective space $\mathbb{P}(\chi)$ is a projective toric variety, and reduces to $\mathbb{C}P^n$ when $\chi_j = 1$ for $0 \leq j \leq n$; otherwise, it is a singular orbifold. In this work we study the algebraic topology of $\mathbb{P}(\chi)$, paying particular attention to its localisation at individual primes p. We identify certain p-specific weight vectors π for which $\mathbb{P}(\pi)$ is homeomorphic to an iterated Thom complex over S^2, and discuss how an arbitrary $\mathbb{P}(\chi)$ may be recovered from its p-specific constituents. We explain how Kawasaki's computations of the integral cohomology ring $H^*(\mathbb{P}(\chi);\mathbb{Z})$ may be expressed in terms of Thom isomorphisms, and extend his results to various generalized cohomology algebras $E^*(\mathbb{P}(\chi))$, including the cases of complex cobordism, Brown-Peterson cohomology, and real K-theory. Wherever possible we give illustrative examples in dimensions 4 and 6, and describe extensions to more general classes of toric and quasitoric orbifolds.

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