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Dynamo models and the flip-flop phenomenon in late-type stars

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ABSTRACT

The 'flip-flop' phenomenon has now been found to occur on several late-type stars: active longitudes, associated with dark regions of the surface, are seen to jump by 180°. A simple non-linear non-axisymmetric mean field dynamo model is described, in which for the first time magnetic behaviour is found that is akin to the observed flip-flop.

Key words: magnetic fields – stars: activity – stars: late-type – stars: magnetic fields – stars: spots.

1 INTRODUCTION

In the last decade or so, a range of observational techniques, both direct and inferential, have contributed to a substantial increase in our knowledge of the magnetic fields that appear to be present at the surfaces of all or nearly all late type stars (see, e.g. Donati, Collier Cameron & Petit 2003 and references therein). One of the more intriguing phenomena is the magnetic 'flip-flop'. This was first reported by Jetsu et al. (1991) for FK Comae, where optical photometry revealed a change by 180° of the predominant 'active longitude'. Active longitudes are typically characterized as cooler regions of the surface, which also possess enhanced magnetic field – initially by analogy with the Sun. Subsequently this flip-flop has been found to recur on FK Comae, and also has been detected on several other stars, mostly in binary systems but including one single late-type dwarf (LQ Hydrae – Berdyugina, Pelt & Tuominen 2002). Besides being of late spectral type, these stars are also relatively rapid rotators, with periods typically of a few days. Berdyugina & Usoskin (2003) also find a similar phenomenon on the Sun

From a theoretical viewpoint, these observations have been hard to explain. These stellar magnetic fields are thought to be generated by a form of dynamo action, in which large-scale magnetic fields are generated and maintained by the action of differential rotation and the statistically non-mirror symmetric convective motions (the $\alpha\omega$ or $\alpha^2\omega$ dynamo). Strong differential rotation can be expected to inhibit the generation of non-axisymmetric fields whilst favouring axisymmetric field production (e.g Rädler 1986). However, rotation laws (generally not of the form $\Omega = \Omega(r)$) can be found for which non-linear solutions with stable non-axisymmetric fields exist (e.g. Rädler et al. 1990; Moss, Tuominen & Brandenburg 1991; Moss et al. 1995). Nevertheless, the steady non-axisymmetric fields investigated in these papers are usually steady in a frame rotating rigidly at approximately the mean stellar angular velocity, and so do not immediately appear capable of driving the observed flip-flop behaviour (Moss 1999 does find weakly oscillatory non-axisymmetric fields in a solar model).

In this Letter a preliminary report of new non-linear dynamo calculations is given, in which for the first time something akin to flip-flop behaviour is found.

2 THE MODEL

The standard alpha-quenched mean field dynamo equations are solved by the method described in Moss et al. (1991) and Moss (1999). Very briefly, the magnetic field is written as a sum of azimuthal Fourier components proportional to $e^{im\phi}$, for $m = 0, \ldots M$. The dynamo equations for each Fourier mode are integrated separately on a meridional $(r,\ \theta)$ grid, and the non-linear term $\alpha_0(r,\theta)\,\boldsymbol{B}/(1+\boldsymbol{B}^2/B_{\rm eq}^2)$ is evaluated in real space, Fourier analysed at each meridional grid point, and the process repeated; $B_{\rm eq}$ is the equipartition field strength (here assumed spatially uniform). The meridional mesh consists of 61 radial and 101 latitudinal points, distributed uniformly over $r_0 R \leqslant r \leqslant R$, $0 \leqslant \theta \leqslant \pi$ respectively, where R is the stellar radius. Useful preliminary results can be found with $M = 1$, and are confirmed with $M = 3$ – energies in modes $m > 1$ are found to be small (see also Moss et al. 1991).

We take the simplest form

$$\alpha_0(r,\theta) = \alpha_* \cos\theta, \qquad (1)$$

with α_* = constant, for the basic alpha coefficient. This form does no more than capture the fundamental symmetry properties of the alpha effect, but is often used in preliminary studies. We are interested in rapidly rotating stars, and so adopt a quasi-cylindrical rotation law in the dynamo regime, which fits smoothly on to an approximately rigidly rotating core at radius $r = r_0$ through a transition region of width order d, centred at $r = r_{\rm c} R$,

$$\Omega(r)/\Omega_0 = 1 + 0.5\left\{1 + {\rm erf}\left[\frac{(r - r_{\rm c})}{d}\right]\right\} ar^2 \sin^2\theta, \qquad (2)$$

where a and Ω_0 are constants, and r is now the dimensionless radius in units of R. Equation (2) is reminiscent of the form used by Bushby (2003), with the significant difference that now the

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surface differential rotation $\propto \sin^2\theta$, as suggested by observations. The turbulent diffusivity is taken to be uniform, $\eta_T = \eta_0 =$ const. For numerical convenience, the uniform rotation term in the definition (2) is omitted in the implementation of the code; axisymmetric solutions are unaffected by this, but the computed non-axisymmetric fields will rotate with an additional angular velocity Ω_0. With this simplification, dynamo solutions depend on the product $a\Omega_0$.

Vacuum boundary conditions are applied at the surface $r = R$, by a matrix multiplication method. The conditions at the inner boundary of the dynamo active region, $r = r_0$, are more arbitrary, and either zero or overshoot conditions are used – tests showed that the results were insensitive to this choice.

After a standard non-dimensionalization in terms of the stellar radius R and diffusion time R^2/η_0, the usual two dynamo parameters $C_\alpha = \alpha_* R/\eta_0$, $C_\omega = a\Omega\, R^2/\eta_0$ are obtained. Note that observed values of a are typically in the range 0.001–0.005, but uncertainties in the value of η_0 allow some freedom of choice for C_ω. For illustration, if $P_{\rm rot} = 1.5$ d, $R \sim {\rm R}_\odot$ (cf. LQ Hydrae) then with $\eta_0 = 10^{12}\ {\rm cm^2\ s^{-1}}$, $a = 0.002$ we obtain $C_\omega \approx 400$. An upper limit to α is $l\Omega_0$, where l is a length-scale in the convection zone. With these parameters, $|C_\alpha| \lesssim 10^4$: this is almost certainly a very generous upper bound.

Gross properties of the solution are monitored by the global energies in the various modes

$$E_i = \int \frac{\boldsymbol{B}_i^2}{2}\, {\rm d}V, \qquad i = 0, \ldots M, \tag{3}$$

and by the corresponding parities $P_i = (E_i^e - E_i^o)/E_i$ (where E_i^e, E_i^o are the energies in the ith mode with even and odd symmetry, respectively, with respect to the rotational equator), $P_0 = -1$ corresponds to S0 – aligned dipole-like; $P_1 = +1$ to S1 – perpendicular dipole-like; etc.

This is undeniably a rather unsophisticated dynamo model, but it appears adequate to demonstrate some key points.

3 RESULTS

3.1 Computations

Here we describe just the solution with $C_\omega = 500$, $C_\alpha = -100$, $r0 = 0.64$, $r_{\rm c} = 0.70$, $d = 0.05$, with $M = 3$. The variation with time of the energies in the field modes $m = 0, \ldots 3$, and the parities of the dominant $m = 0$ and $m = 1$ components are shown in Fig. 1. The field undergoes a regular, quasi-sinusoidal oscillation in magnitude. The $m = 1$ part has parity $P_1 \approx 0$, whilst P_0 oscillates strongly. In Fig. 2 a snapshot of the contours of constant $|\boldsymbol{B}|$ over the surface is shown. The non-axisymmetric features are strongly concentrated in one hemisphere, to latitudes around 60° S–90° S (this is consistent with $P_1 \approx 0$).

Behaviour of this general type was found for a relatively small range of parameters (covering a factor of about 2 in C_ω and $-125 \lesssim C_\alpha \lesssim -20$). For orientation, with $C_\omega = 500$, the marginal values of C_α for excitation of the $m = 0$ and $m = 1$ linear dynamo modes are −11.6 and −13.4 respectively. For large enough $|C_\alpha|$ and small enough C_ω, the dynamo becomes steady ($\sim\alpha^2$ system). For large enough C_ω, the non-axisymmetric field is not excited (differential rotation too strong, e.g. Rädler 1986).

3.2 Analysis

The raw results are analysed as follows. At a given time, the modulus of the magnetic field at the immediately sub-surface radial mesh

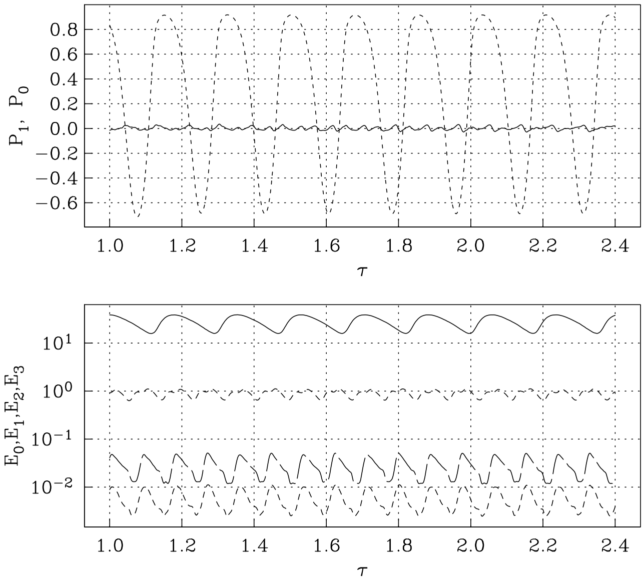

Figure 1. The lower panel shows the variation in time of the energies in the modes $m = 0$, 1, 2 and 3 (from top to bottom, respectively). The upper panel displays the parity of the $m = 0$ (broken) and $m = 1$ (solid) parts of the field.

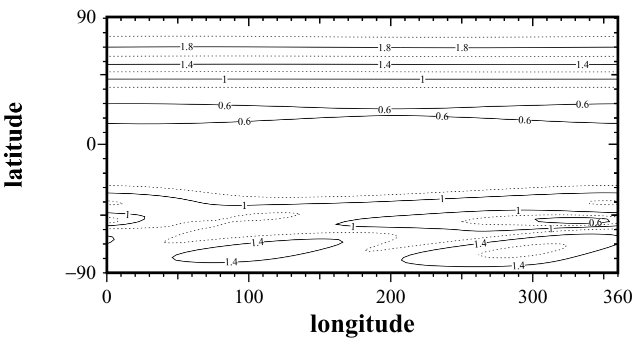

Figure 2. Contours of equal field strength $|\boldsymbol{B}|$ over the surface, at time $\tau = 2.4$.

point is integrated from pole to pole at a range of longitudes ϕ, yielding $\overline{B(\phi)} = \int_0^\pi |\boldsymbol{B}| \sin\theta\, {\rm d}\theta$. From this the longitude, $\phi_{\rm m}$, with maximum $\overline{B(\phi)}$ can be found. For any computation, a time series of $\phi_{\rm m}(t)$ can be constructed. Non-axisymmetric field structures naturally rotate (with respect to the background angular velocity Ω_0), so $\phi_{\rm m}$ for a steady, purely non-axisymmetric, field can be expected to change monotonically with time. Here, of course, we have a combination of a (weakly) time-dependent non-axisymmetric field and an oscillating axisymmetric field.

In Fig. 3 $\phi_{\rm m}(t)$ is shown for the model described in Section 3.1. The underlying repeated sawtooth pattern demonstrates such a field rotation as described above, but at intervals a jump in $\phi_{\rm m}$ is observed. Fig. 1 shows five major jumps, at $\tau \approx 2.18$, 2.23, 2.27, 2.31 and 2.35, all of about 180°. There are also three less regularly spaced jumps of $\lesssim 90°$. The jumps from 360° to 0° are, of course, unphysical. The circa 180° jumps might be interpreted as prototype flip-flops. (When the computation is restricted to $M = 1$, the gross properties of the solution are very similar, but the minor jumps in ϕ_m are absent and the periodicity is evident.)

Because of the smaller-scale structure in the field, it is possible for Fig. 3 to show rather trivial events – or at least it is unclear that they

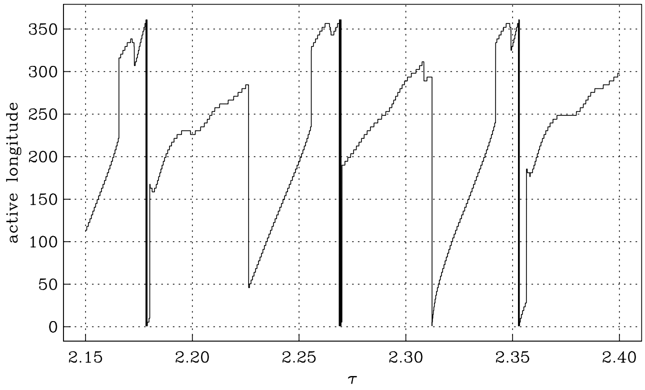

Figure 3. The value of ϕ_m (one measure of the 'active longitude') as a function of time.

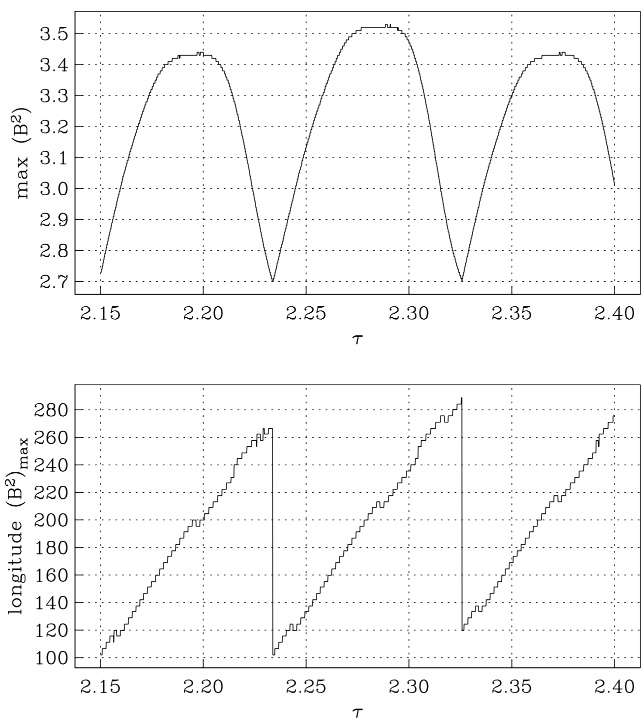

Figure 4. The lower panel shows the longitude of the region with maximum surface field strength in the southern hemisphere as a function of time. In the upper panel, the variation of the surface value of $\boldsymbol{B}^2_{\rm max}$ in this hemisphere is given. The small-scale discreteness is a consequence of the sampling used.

would all be detectable. Thus in Fig. 4 we show also the longitude of the maximum of $|\boldsymbol{B}|$ in the southern hemisphere. Apart from any illustrative function, this might be more related to what is seen from an angle that observes mostly the southern hemisphere. Now jumps are rather cleaner, and all near 170°. Only some of the features in Fig. 3 appear here.

4 DISCUSSION

Contrary to some previous statements in the literature, it is in principle quite easy to see how a combination of simple dynamo modes can result in an alternation of active longitudes, if interpreted as maxima of surface field distributions. Consider for example an oscillating axisymmetric quadrupole (S0 field) and a constant perpendicular dipole (S1 field) with, for example, the field concentrated at high latitudes (as found in earlier studies of non-linear non-axisymmetric fields). Suppose in some frame the S1 field is directed outwards with maximum strength at longitude 0°. When the S0 field is directed outward at high latitudes, the latitudinally integrated fields will reinforce near longitude 0°, and this will be the active longitude – consider Fig. 5. Half a period later, the S0 field will have reversed, and the active longitude will be at 180° – see Fig. 5 with the arrows on the S0 field reversed. *It is not necessary for the non-axisymmetric field to oscillate* (in contrast to the impression given by Berdyugina et al. 2002, although in the solution presented in Section 3 the non-axisymmetric field is indeed unsteady). A similar argument can be constructed with an oscillatory A0 field and a steady mixed parity non-axisymmetric field that is concentrated in one hemisphere (A1 + S1), or with oscillating A0 and steady A1 fields.

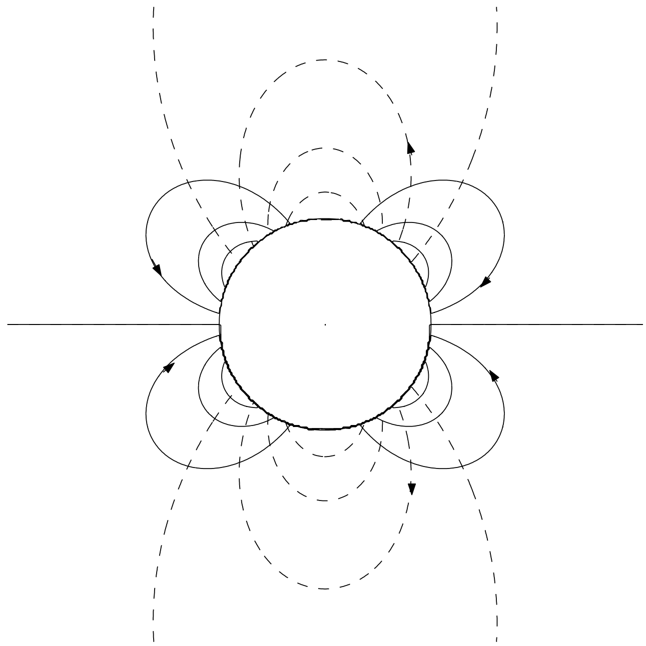

Figure 5. A sketch of superimposed magnetic fields of S0-type (solid) and S1-type (broken). With the fields directed as indicated, the latitudinally averaged field has a maximum at longitude 0° (on the right). When the direction of the S0 field is reversed, the maximum is at longitude 180°.

In the solution discussed here, there are no pure parity fields, and the situation is clearly more complex than in the the illustrative examples just given. This solution does not have, for example, a simple S0 + S1 structure, rather a (weakly) oscillating non-axisymmetric field with approximately A1 + S1 structure, and an axisymmetric field with oscillating parity whose structure is for part of the time near A0. The non-axisymmetric field pattern rotates both with respect to the central rotation speed Ω_0 and the surface equatorial speed $(1 + a)\Omega_0$. This is qualitatively consistent with the observed behaviour of the active longitudes by Korhonen et al. (2001), and the computed major flip-flops are nearly, but not exactly, periodic, recurring after O(100) rotation periods. Again this is loosely consistent with the observed behaviour of FK Comae, although no attempt was made to tune the model to fit observations of this star. The smaller jumps in ϕ_m do not appear to have observational counterparts.

Note also that here the pole-to-pole integrated field has been considered as a proxy for activity. In the numerical model the two hemispheres do not, in general, behave in synchrony, and so for a model viewed from small inclination i, an appropriate weighting for the relevant hemisphere would have to be applied to deduce the 'observed' behaviour. This will be relevant if efforts are made to model specific observed stars. A preliminary alternative measure is presented in Fig. 4.

The limited parameter range in which flip-flop behaviour is found is not necessarily a challenge. It is at present unproven that more than a minority of stars exhibit this behaviour. On the other hand, it is possible that a model in which the Lorentz force is allowed to react back on the rotation field may quite naturally adjust itself to a 'favourable' configuration for a wider range of parameters (cf. Moss et al. 1995).

The existence of significantly oscillatory non-axisymmetric fields appears relatively novel (but see also Moss 1999): an obvious explanation is that these oscillations are driven by the oscillations of the axisymmetric field via the mediation of the alpha-quenching mechanism.

As observed in Section 1, a weak flip-flop solar behaviour has also been reported. The Sun has a predominantly A0 field, with a weak and rather ill-defined non-axisymmetric component (a preliminary model was published by Moss 1999). Thus any solar flip-flop may result from a somewhat different mechanism. Of course, the solar interior rotation law is quite different to (2).

The reason that this phenomenon has not been noted previously seems to lie in the very limited number of published non-linear non-axisymmetric stellar dynamo models. The author knows only that of Rädler et al. (1990) and several papers by himself and collaborators (see e.g. Moss et al. 1995). These studies all use rather idealized rotation laws that are now believed to be unphysical. There is also the solar model of Moss (1999). This present Letter appears to be the first to describe non-linear non-axisymmetric models with quasi-cylindrical rotation laws, appropriate to rapidly rotating stars.

5 CONCLUSIONS

The preliminary results presented here provide for the first time a theoretical basis for understanding the flip-flop in the context of mean field dynamo models. Several properties of the model appear broadly consistent with observed behaviour. As stressed before, the model is very simple, and it is easy to think of plausible improvements. Experience shows that parity in particular is sensitive to relatively small changes in the way the model is set up. In the present context, this could affect the behaviour of significant features of the solutions.

It is also true that the non-axisymmetric field is rather weak in the model discussed, and it is quite probable that a modification of the model could give a stronger non-axisymmetric field and associated active longitude jumps. Nevertheless, it is reasonable to regard this as a prototype for explaining the 'flip-flop' phenomenon.

More abstractly, the existence of an oscillating non-axisymmetric field coexisting stably with an axisymmetric field is a little-reported phenomenon. Further, more comprehensive, results will be presented in a later paper.

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