Magnetic fields in fully convective M-dwarfs: oscillatory dynamos versus bistability

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ABSTRACT
M-dwarfs demonstrate two types of activity: (1) strong (kilogauss) almost axisymmetric poloidal magnetic fields; and (2) considerably weaker non-axisymmetric fields, sometimes including a substantial toroidal component. Dynamo bistability has been proposed as an explanation. However, it is not straightforward to obtain such a bistability in dynamo models. On the other hand, the solar magnetic dipole at times of magnetic field inversion becomes transverse to the rotation axis, while the magnetic field becomes weaker at times far from that of inversion. Thus, the Sun resembles a star with the second type of activity. We suggest that M-dwarfs can have magnetic cycles, and that M-dwarfs with the second type of activity can just be stars observed at times of magnetic field inversion. Then the relative number of M-dwarfs with the second type of activity can be used in the framework of this model to determine parameters of stellar convection near the surface.

Key words: magnetic fields – Sun: activity – stars: activity – stars: late-type – stars: magnetic field.

1 INTRODUCTION
The efforts of many observing teams over several decades have provided rich data concerning magnetic activity in stars of various spectral classes. However, in many cases, the general form of the activity of stars of a given spectral class remains debatable. In particular, observations find two distinct magnetic topologies for M-dwarfs: (1) strong (kilogauss) almost axisymmetric poloidal magnetic fields, and (2) considerably weaker non-axisymmetric fields, sometimes including a substantial toroidal component (Morin et al. 2010). Stars with different topologies can have roughly the same rotation rates, mass, and other parameters. This finding has been interpreted as dynamo bistability, i.e. as two different regimes of dynamo action being stable for the same set of stellar parameters (Morin et al. 2011a, b; Gastine et al. 2013). However, two stable regimes of a dynamo in the same parameter range seem not to be very probable (cf. e.g. Rädler et al. 1990; Moss & Sokoloff 2009). On the other hand, Chabrier & Küker (2006) suggest that fully convective stars host a non-axisymmetric global dynamo (such a possibility was also suggested in a slightly different context by, e.g., Barker & Moss 1994). This dynamo is steady (equatorial dipole drifting in longitude) and therefore showing no reversals. This model relies on the observational fact (Burnes et al. 2005) supported by numerical simulations (Küker & Rüdiger 2005) that differential rotation in fully convective stars is very small. As a result, Chabrier & Küker (2006) neglect differential rotation and considered an \( \alpha \)2 dynamo.

The efficiency of differential rotation in winding up magnetic fields can be estimated by the dimensionless dynamo number

\[
C_\Omega = \frac{\Delta \Omega H^2}{\eta_t},
\]

where \( \Delta \Omega \) is the angular velocity variation within the convection zone, \( H \) is the convection zone thickness and \( \eta_t \) is the eddy magnetic diffusivity. Differential rotation modelling suggests that the decrease in \( \Delta \Omega \) with decreasing temperature can be compensated by a simultaneous decrease in \( \eta_t \), so that \( C_\Omega \) actually increases in cooler stars (Kitchatinov & Olemskoy 2011; Kitchatinov 2013). This means that the M-dwarfs have small but very efficient differential rotation and it is quite probable that they host oscillatory, axisymmetric mean-field dynamos.

Taking all this into account, it seems reasonable to consider another possible explanation of the M-dwarf activity phenomenon. We suggest that magnetic cycles in the form of oscillatory axisymmetric fields are present within the population of M-dwarfs, and that the stars that are observed to have weak non-axisymmetric fields are observed at epochs of reversal, with the strong axisymmetric dipoles being present at the analogues of solar minima. There are strong observational indications for a difference in magnetic topologies between fully convective stars and stars with convective envelopes, with magnetic fields of fully convective stars being dominated by...
axisymmetric poloidal configurations (Gregory et al. 2012). Nevertheless, an analogy with the dynamics of the (relatively weak) solar poloidal field is possible and can be useful. Note that the solar activity is determined mainly by the toroidal magnetic field. There is a time lag between the cyclic oscillations of toroidal and poloidal magnetic fields, so that the solar magnetic dipole is strong during the minima of solar activity. A straightforward verification of this explanation could be performed by monitoring of a sample of M-dwarfs over times exceeding the expected period. Estimates in the next section suggest however that magnetic cycles in M-dwarfs can last for several decades. If M-dwarfs do have cycles, the cycles could therefore be substantially longer than that of the Sun, while at the moment only a three-year monitoring (2006–2009) is available (Morin et al. 2010).

A straightforward objection to the last explanation could be that solar mean-field dynamo models driven by differential rotation and mirror-asymmetric turbulence (αΩ dynamos) give axisymmetric mean magnetic fields, and the magnetic dipole has to vanish during its inversion, and be parallel to the rotation axis between its reversals. Recent progress in understanding solar observations and the solar dynamo (Moss, Kitchatinov & Sokoloff 2013a; Moss et al. 2013b) has provided a new understanding of the reversals of the solar magnetic dipole. This allows us to elaborate the idea under discussion quantitatively, and to suggest a way to verify it that does not ultimately require a long-term monitoring programme (which of course still remains highly desirable).

2 OSCILLATORY DYNAMOS IN M-DWARFS

Taking into account the inherent uncertainties and arbitrariness of dynamo modelling for wide classes of stars, and that it is far from clear how generic any particular model (set-up, choice of parameters, etc.) can be, nevertheless we performed an exploratory modelling to establish the possibility of oscillatory dynamos existing in M-dwarfs, as follows.

We computed the differential rotation for a star with \( M = 0.3 \, M_\odot \), rotating with a period of 10 d, using the numerical mean-field model of Kitchatinov & Olemskoy (2011). The result is shown in Fig. 1. Such a star is fully convective, but the model requires an (in this case artificial) inner boundary of the convection zone to be imposed at \( r = 0.1 \, R_{\star} \). The model does not prescribe eddy transport coefficients but estimates them from the entropy gradient, so that the \( C_\Omega \) parameter of equation (1) can also be estimated. Taking the turbulent Prandtl number,

\[
P_m = \frac{v_T}{\eta_T},
\]

Figure 1. Isorotational curves (left) and surface rotation frequency versus latitude (right) for the reference M-dwarf model.

to have the value unity gives \( C_\Omega \simeq 290 \), which is not small, in spite of the small differential rotation in Fig. 1. The eddy viscosity \( v_T \simeq 1.2 \times 10^{11} \, \text{cm}^2 \, \text{s}^{-1} \) used in this estimate is taken at the middle radius \( r = R_{\text{rad}}/2 \) (\( R_{\text{rad}} = 212 \, \text{Mm} \)). Thus, the diffusive time is \( \sim 100 \, \text{yr} \). If the cycle time is close to the diffusion time, as it is for the Sun, very long cycles can be expected. The circulation time for the meridional flow is also long, \( \sim 60 \, \text{yr} \) (the typical flow velocity is \( 10 \, \text{cm} \, \text{s}^{-1} \) except in the thin surface boundary layer, where it is about \( 4 \, \text{m} \, \text{s}^{-1} \)). Thus, the cycle time for an advection-dominated dynamo will also be long. We use these estimates to obtain the reference values for the dynamo governing parameters for M-dwarfs and experiment with numbers around these reference quantities.

We used a kinematic model of an \( \alpha \Omega \) dynamo to compute the threshold values of the dimensionless parameter,

\[
C_\alpha = \frac{\alpha \Omega R_{\text{rad}}}{\eta_T},
\]

where \( \alpha \Omega \) is the amplitude of the \( \alpha \)-effect for the onset of dynamo instability of magnetic field modes of different equatorial and axial symmetries. The model solves the eigenvalue problem for the mean-field dynamo equations (cf., e.g., Krause & Rädler 1980) numerically by applying a grid point method in radius and a latitudinal expansion in Legendre polynomials. Vacuum conditions were applied at the outer boundary and at the inner boundary the solution was matched to a perfect conductor. Our computations were performed on a uniform radial grid of 301 points, and with 40 Legendre polynomials in the latitudinal expansion.

Figs 2 and 3 demonstrate the results for the simplest model with uniform diffusion, \( \alpha = \alpha_0 \cos \theta \), i.e. uniform with radius and a \( \cos \theta \) dependence on colatitude \( \theta \), and diffusion and the \( \alpha \)-coefficient are isotropic. The eddy viscosity is estimated by the differential rotation code. We take the magnetic Prandtl number of equation (2) as a free parameter. Fig. 2 shows that oscillatory axisymmetric global modes are preferred (need smaller \( C_\alpha \) for excitation) for \( P_m > 2 \). These modes have oscillation periods of order 100 yr.

We note in passing that naively the rotation law illustrated in Fig. 1 might be expected to give predominantly radial field migration. However, Fig. 3 suggests rather a weak poleward migration. The model is only slightly supercritical and strong statements should perhaps be avoided. For our model to be viable, we only need to demonstrate that plausible solutions in which the poloidal field oscillates can be found.

We conclude from this limited modelling that an oscillatory dynamo model with solar-type behaviour can be considered as a viable option, at least at the present level of investigation.

Figure 2. A dynamo model for an M-dwarf: left – marginal values of \( C_\alpha \) (3) for axisymmetric modes (A0) and non-axisymmetric \( m = 1 \) modes (A1) of dipolar parity, dots indicate steady axisymmetric modes and the solid line shows the oscillatory modes, right – the corresponding oscillation periods of axisymmetric modes.
3 REVERSALS OF THE SOLAR DIPOLE AS A PARADIGM FOR REVERSALS IN M-DWARFS

We start by noting that many solar observers (e.g. Antonucci 1974; Zhukov & Veselovsky 2000; Livshits & Obridko 2006) have reported that the solar magnetic dipole does not vanish during the reversal. Recently, De Rosa, Brun & Hoeksema (2012) presented a comprehensive data sample for the two last solar activity cycles which convincingly demonstrate that this is indeed the case.

Moss et al. (2013a) suggested how to resolve this apparent contradiction between expectations from dynamo modelling and observation. The point is that a mean-field dynamo model deals with mean magnetic field and the averaging is performed over an ensemble of convective velocity cells, while the observational magnetic dipole data refer to large-scale magnetic field. Both quantities coincide for an infinitely large ensemble of convective cells, but in practice the number of cells is only moderately large (N = 10^4 is a crude relevant estimate, see Moss et al. 2013a for details). Because the convective cell ensemble contains a not extremely large number of cells, large-scale fluctuations of magnetic field arise which yield a fluctuating component δd of the solar magnetic dipole d, of order

$$\frac{b}{B} N^{-1/2} \left(\frac{B_P}{B_T}\right) .$$

Here, b is the rms value of small-scale magnetic field, i.e. the magnetic fluctuations, B is the typical value of the mean magnetic field which is determined mainly by the toroidal magnetic field B_T, and the factor B_P/B_T takes into account that the magnetic dipole moment is determined by the poloidal magnetic field B_P. Moss et al. (2013a) demonstrated that the estimate for the solar magnetic dipole at the epoch of inversion, based on equation (4) and the available information about the relevant solar parameters, is consistent with this picture.

Moss et al. (2013b) compared this scenario with observational data more systematically, and found that the interval during which the fluctuating part of the magnetic dipole is larger than the part determined by the mean magnetic field is about four months, i.e. about 3 per cent of the solar magnetic cycle. In order to demonstrate the mechanism more explicitly, we show in Fig. 4 the migration of the dipole from one solar pole to the other in an example taken from

![Figure 4. Trajectory of the direction of the solar dipole during a reversal of the solar magnetic field.](image-url)
4 RESCALING THE SOLAR EXAMPLE

Assuming that the magnetic activity of M-dwarfs is more or less similar to that of the Sun, we can use equation (4) to determine the relative time $\delta t/T$ during the magnetic activity cycle during which the magnetic dipole is determined by magnetic fluctuations and is strongly inclined to the rotation axis. Because the scenario assumes that at that time the axisymmetric dipolar field is minimal, we conclude that observations taken at that epoch would identify this star as exhibiting the second type of activity.

If we consider a random sample of M dwarfs, then the value $\delta t/T$ immediately gives the relative number $\delta n/N$ of M-dwarfs in the sample which demonstrates the second type of activity.

The temporal evolution of the axisymmetric solar magnetic dipole is approximately sinusoidal, see Moss et al. (2013a), i.e. much simpler than the evolution of the mean sunspot number. Assuming that the same is valid for M-dwarfs, and taking into account equation (4), we can say what physical parameters are required, e.g. $\delta n/N$, to get a satisfactory correspondence with the observational data. In particular, if we want to explain that about 30 per cent of M-dwarfs demonstrate the second type of activity, we have to assume that the number of convective cells at the surface of M-dwarfs is about two order of magnitudes lower than near the surface of the Sun, i.e. $10^2$ instead of $10^4$. (Given the much greater relative depth of the convection zone in M-dwarfs compared to the Sun, an increase in the size of convection cells is not implausible. For example, Schwarzschild (1975) suggests that the horizontal scale is determined by scale heights deep in the convective region, which here encompasses the whole star. Observations of the convective giant Betelgeuse suggest the presence of a few, very large, cells at the surface, see e.g. Chiavassa et al. (2010), and Tremblay et al. (2013) suggest that there is a resemblance between the nature of convection in M-dwarfs and giant stars.) Then, equation (4) reads that $\delta d/d$ increases by a factor of 10 in comparison with the Sun and because the oscillations of the dipole strength are near to sinusoidal, the time at which fluctuations are stronger than the mean value of the dipole grows by a factor of $10^2$, and now becomes about 1/3 of the cycle duration. As a result, the relative number of M-dwarfs which demonstrate the second type of activity increases by a factor of 10, giving 30 per cent.

5 DISCUSSION AND CONCLUSIONS

We have suggested a scenario which explains why observations identify two types of magnetic activity in M-dwarfs. We propose that an M-dwarf can either be observed at a time far from that of magnetic field inversion (giving the first type of activity), or near the inversion (giving the second type).

A quantification of the scenario under discussion can be performed on the basis of equation (4) by determination of the percentage of M-dwarfs that demonstrate the second type of activity. We stress that such quantification does not require an effort consuming long-term monitoring of M-dwarf activity. Of course, such a monitoring remains highly desirable for further understanding of stellar magnetic activity.

Even a crude estimate of the relative number of M-dwarfs with the second type of activity will provide in our framework a perspective for monitoring activity in M-dwarfs. If, say, this number were about 10 per cent, then it would only be necessary to perform monitoring during 1/10 of a stellar cycle in order to observe a transition from one activity type to the other one in a given object. Another problem of interest would be to determine how this relative number varies between solar-type stars and M-dwarfs. Indeed, any star with periodic dynamo driven large-scale fields might be expected to display similar behaviour, but whether this could be detected by current observational techniques is another matter.

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REFERENCES


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