1. Show that if \( L_1, L_2 \subseteq A^* \) and \( L_1, L_2 \in \text{Time}(n^r) \) then \( L_1 \cup L_2, L_1 \cap L_2, A^* \setminus L_1 \in \text{Time}(n^r) \).

2. Show that the language
\[
L_e = \{ \sigma \in \{0, 1\}^* \mid \sigma \text{ contains the same number of 1's as 0's} \} \subseteq \{0, 1\}^*
\]
is in \( \text{Time}(n|n|) \), where \( |n| \) denotes the length of the binary representation of \( n \).

3. Let \( \sigma_M \) code a TM \( M \) which runs in time \( T(n) \) and space \( S(n) \). Show that there exist constants \( \delta \) and \( \lambda \) (depending only on \( \sigma_M \)) such that, when the 3-tape universal TM \( \mathcal{U} \) from Section 1.14 is run with input \( \sigma_M 111111 \tau \), it halts within \( \delta T(|\tau|) \) steps and visits no more than \( \lambda S(|\tau|) \) squares.

4. Show that if \( M \) is a TM accepting \( L_e \) and running in time \( T(n) \) for some monotonically increasing function \( T(n) \), then there is a constant for \( \alpha > 0 \) such that for all sufficiently large \( n \) we have \( T(n) \geq \alpha n \log_2 n \).

[Hint: considering crossing sequences at interfaces
\[
1^r 0^{m-r}(10)^i(10)^{n-2m-i}1^{m-r}0^r,
\]
where \( n \) is even and \( m \) is the integer part of \( n/3 \). Here the notation \( (10)^i \) means the word \( 101010 \ldots 10 \) obtained by repeating \( 10 \) \( i \) times.]

5. Show that the language
\[
\{ a_1@a_2@ \ldots @a_n@b \mid a_1, \ldots, a_n, b \in \mathbb{N} \text{ and } a_1 + a_2 + \ldots + a_n = b \}
\]
is in \( \mathcal{P} \).

6. Show that the language
\[
PRIMES = \{ \pi \mid n \in \mathbb{N} \text{ and } n \text{ is prime} \}
\]
is in \( \mathcal{NP}^c \). You may assume that the language
\[
\{ \pi@b@c \mid a, b, c \in \mathbb{N} \text{ and } ab = c \}
\]
is in \( \mathcal{P} \).
7. Show the inclusions in
   \[ \mathcal{P} \subseteq \mathcal{N}\mathcal{P} \subseteq PSpace \subseteq \bigcup_{p(x) \in \mathbb{N}[x]} Time(2^{p(n)}). \]

8. Show that \( \mathcal{P} \subseteq Time(2^n) \).

9. Show that if \( L_1, L_2 \subseteq A^* \) and \( L_1, L_2 \in \mathcal{N}\mathcal{P}^c \) then \( L_1 \cap L_2, L_1 \cup L_2 \in \mathcal{N}\mathcal{P}^c \).

10. Suppose that \( L_1 \subseteq A_1^*, L_2 \subseteq A_2^*, L_2 \neq A_1^*, f : A_1^* \rightarrow A_2^*, f \) is polynomial time computable and for all \( \sigma \in A_1^* \),
   \[ \sigma \in L_1 \iff f(\sigma) \in L_2. \]

   Show that there is a polynomial time computable function \( g : A_3^* \rightarrow A_1^* \) such that for all \( \sigma \in A_1^* \),
   \[ \sigma \in L_1 \iff g(\sigma) \in L_2. \]

11. Let \( p(x) \in \mathbb{N}[x] \) and let \( f : A^* \rightarrow \{0, 1\}^* \) be defined by \( f(\sigma) = 1^{p(|\sigma|)} \).

   Show that \( f \) is polynomial time computable.

12. Prove Theorem 18 parts (i), (iii) and (iv).

13. Construct \( L_1, L_2 \subseteq \{0, 1\}^* \), with \( L_1, L_2 \neq \emptyset \), such that neither \( L_1 \preceq_p L_2 \) nor \( L_2 \preceq_p L_1 \). [Harder than your average problem.]

14. Show that \( \text{KNAPSACK} \in \text{Space}(n) \).

15. Show that the relation \( \equiv_p \) on languages defined as
   \[ L \equiv_p L' \iff L \preceq_p L' \land L' \preceq_p L \]
   is an equivalence relation.

16. Show that if \( L \subseteq A^* \) then there is \( L' \subseteq \{0, 1\}^* \) such that \( L \equiv L' \).

17. Show that if \( \mathcal{N}\mathcal{P} = PSpace \) then \( \mathcal{N}\mathcal{P} = \mathcal{N}\mathcal{P}^c \).