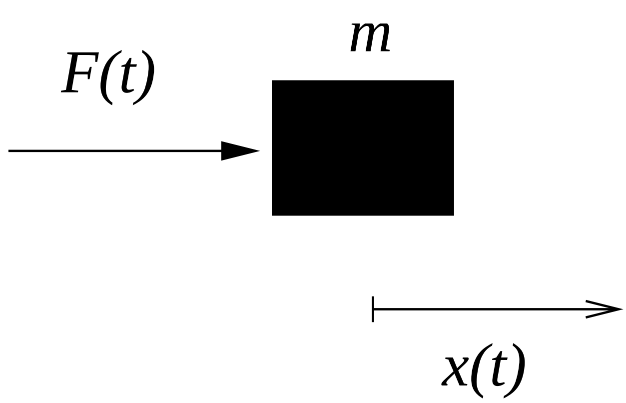
Everything you always wanted to know about mechanical oscillators but were afraid to ask

- The first half of MATH10222 is not directly concerned with mechanics.
- However, mechanical systems provide nice illustrations of many of the phenomena that we have discussed (or will discuss) in a more abstract mathematical setting.

I. Newton's law for one-dimensional motion

- In words: "*The sum of all forces acting on a particle of mass m is equal to its mass times its acceleration*"

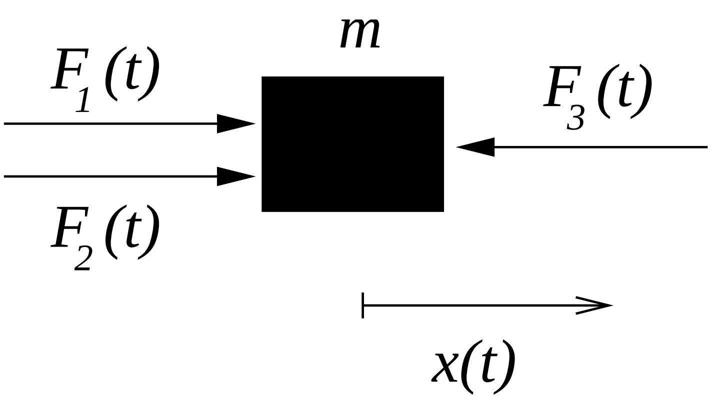


- Or, written as an equation:

$$m\frac{d^2x}{dt^2} = F(t)$$

I. Newton's law for one-dimensional motion (cont.)

- Here's an example with multiple forces



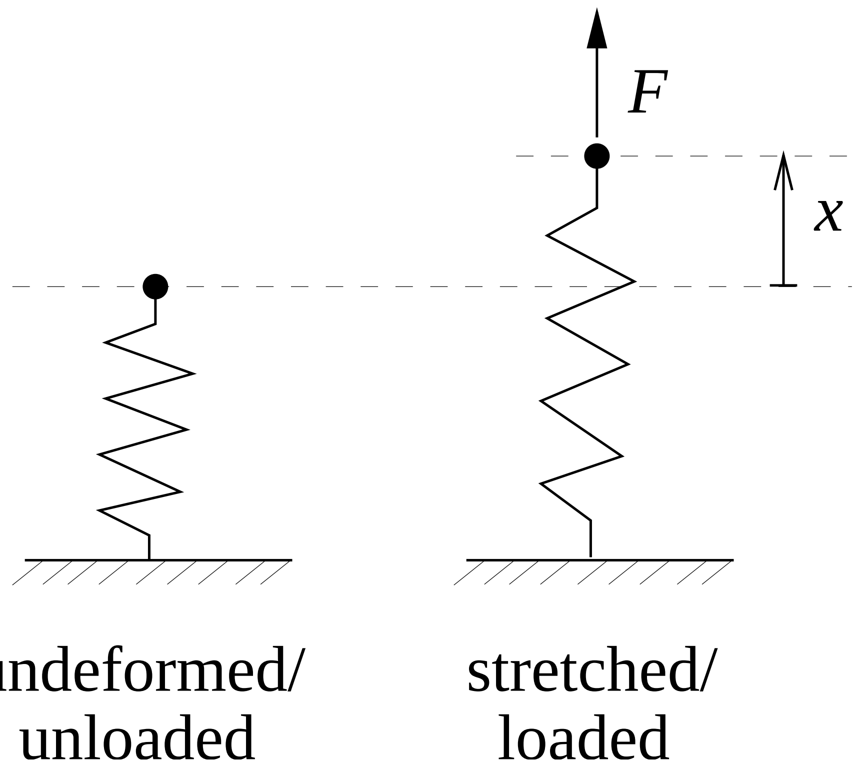
- In this case Newton's law becomes:

$$m\frac{d^2x}{dt^2} = F_1(t) + F_2(t) - F_3(t)$$

- Note the direction of the forces!

II. (Linearly) elastic springs

- Observation: When a spring is loaded by a force, F, its length increases by a certain amount, x, say.



- For a linearly elastic spring we have

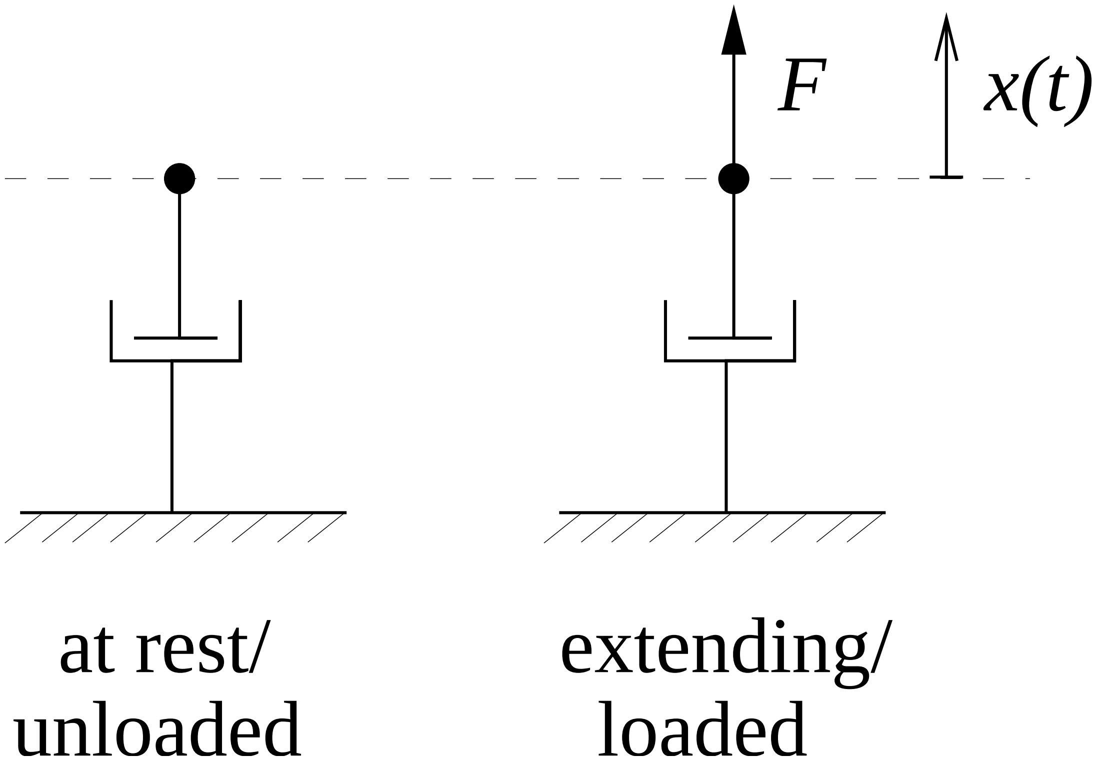
$$F = c\,x$$

where c is the "spring constant", a measure of its stiffness.

- Thus c indicates how strongly the spring resists its *static* extension.

III. (Linear) dampers

- Observation: When a damper is loaded by a force F its length increases at a rate dx/dt:



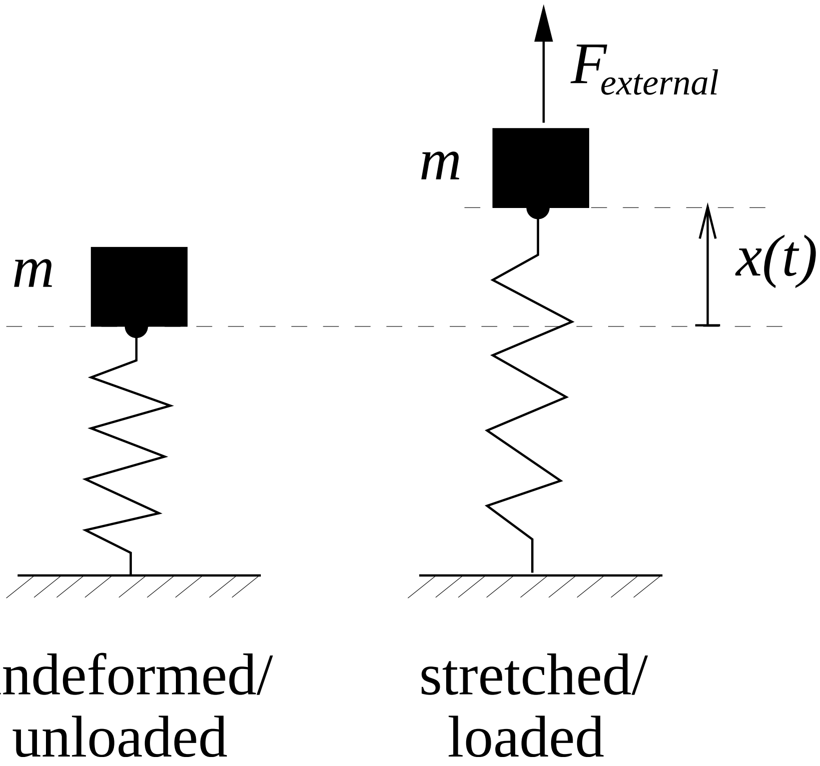
- For a linear damper we have

$$F = k \, \frac{dx}{dt}$$

where k is the "damping constant", a measure of how strongly the damper resists its *dynamic* extension.

IV. Putting it all together: "Action = Reaction"

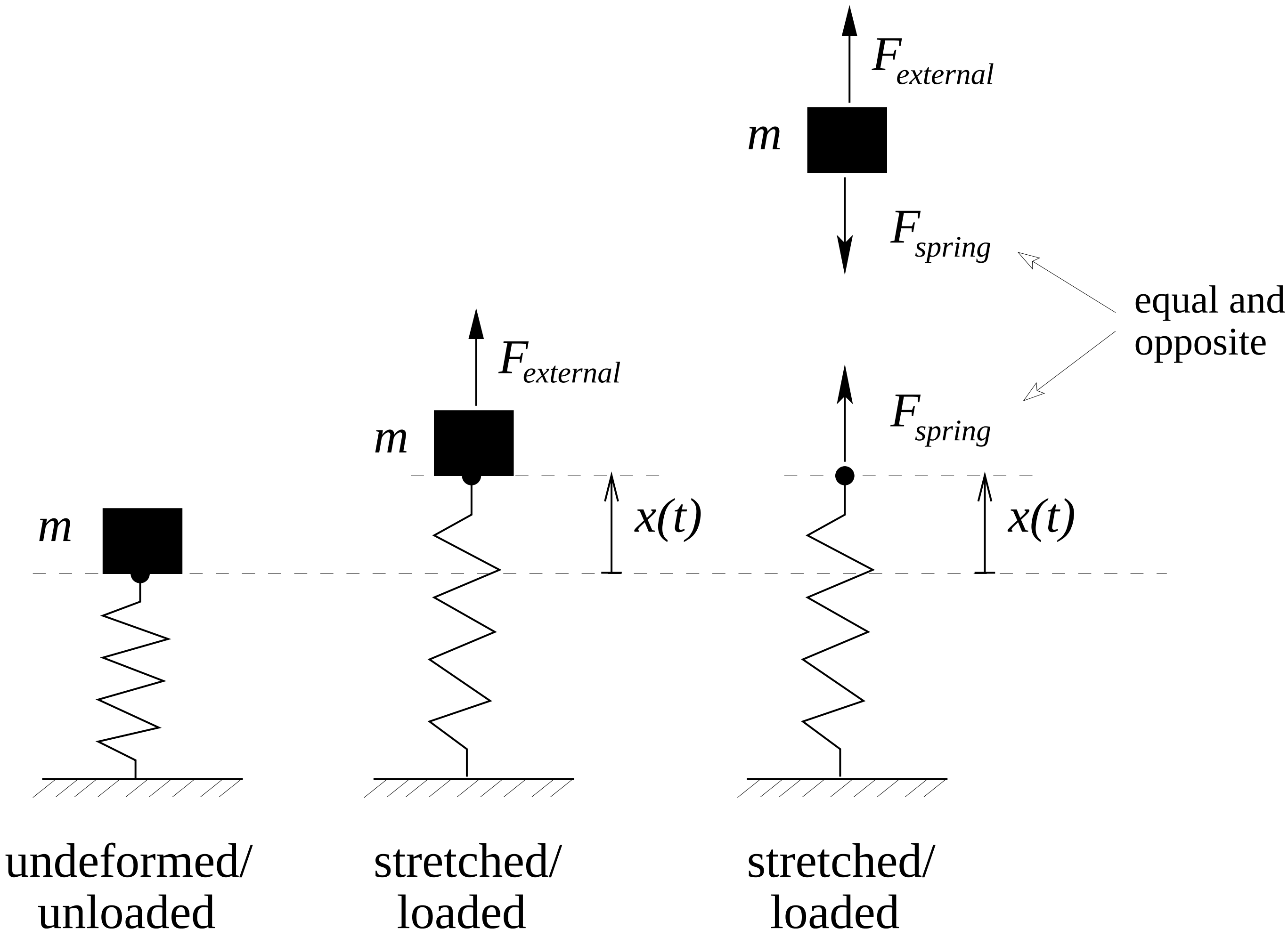
- Here is a mass m, attached to a spring of stiffness c, and loaded by a force, $F_{external}$.



- What is the equation of motion for the mass?
- Write down Newton's law for the mass.
- $\Longrightarrow$ What forces act on the mass?

IV. Putting it all together (cont.)

- "Action = Reaction": The spring pulls the mass and mass pulls the spring (in the opposite direction, obviously!):



- Thus Newton's law states

$$m\frac{d^2x}{dt^2} = F_{external} - F_{spring},$$

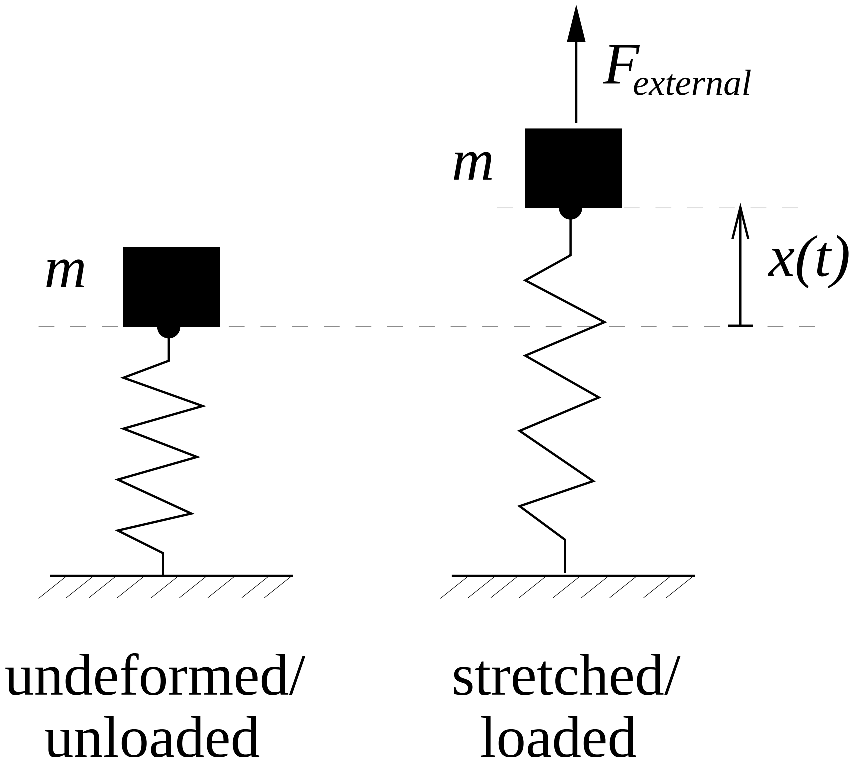
or, using what we've just learned about linear springs:

$$m\frac{d^2x}{dt^2} = F_{external} - cx.$$

IV. Putting it all together (cont.)

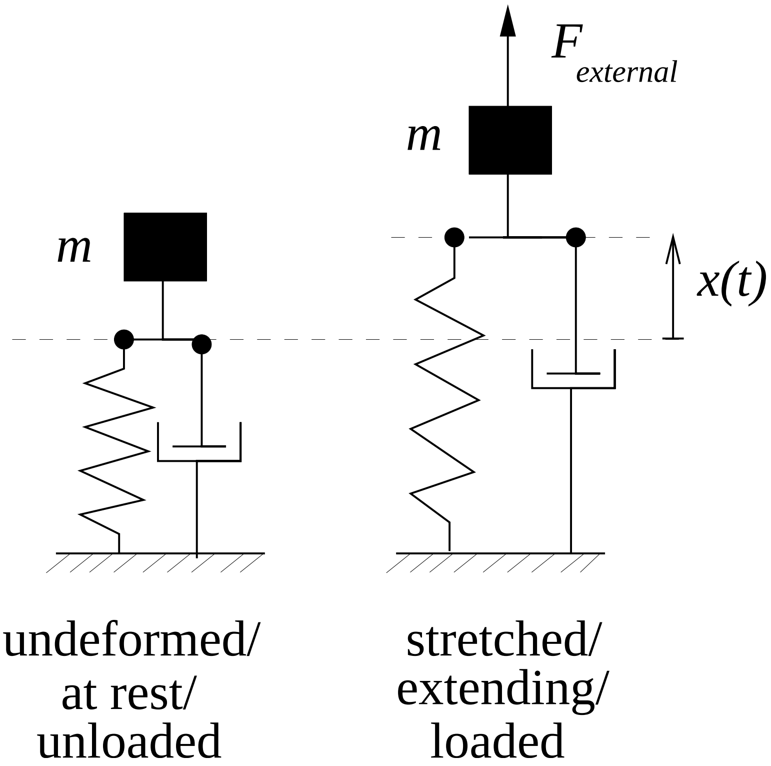
- Rewrite to the standard form of a second-order ODE for $x(t)$:

$$m\frac{d^2x}{dt^2} + cx = F_{external}.$$



Exercise: Try it for yourself

- Here is a mass m, attached to a spring of stiffness c, and a damper (damping constant k), loaded by a force $F_{external}$.



- Show that the equation of motion for the mass is

$$m\frac{d^2x}{dt^2} + k\frac{dx}{dt} + cx = F_{external}$$