

MATH10222: SOLUTIONS TO EXAMPLE SHEET¹

I

1. Classifying ODEs

- (a) **Nonlinear** because of the squared u in the second term on the LHS.
Non-autonomous because the independent variable, x , appears explicitly (on the RHS).
First order because the highest derivative of the unknown function with respect to the independent variable is $u'(x)$ – a first derivative.
- (b) **Linear** because the ODE is linear in the unknown function, u , and its derivatives.
Non-autonomous because the independent variable, t , appears explicitly (in the term multiplying $u(t)$ on the LHS).
Fourth order because the highest derivative of the unknown function with respect to the independent variable is d^4u/dt^4 – a fourth derivative.
- (c) **Nonlinear** because the ODE involves the sin of the unknown function, $\theta(t)$.
Autonomous because the independent variable, t , does not appear explicitly.
Second order because the highest derivative of the unknown function with respect to the independent variable is $\ddot{\theta}$ – a second derivative.

2. Properties of ODEs

- (a) False. $(\frac{d\phi}{ds})^2 = 2s\phi$ is a first order ODE. The highest derivative of the unknown function with respect to the independent variable is $\frac{d\phi}{ds}$ – a first derivative.
- (b) False. u'' and u' are evaluated at different values of the independent variable, namely at x and at $x - 1$, respectively, so $u''(x) + u'(x - 1) = 1$ is not an ODE. (It's a delay differential equation – a different beast altogether).
- (c) False. $\frac{dx}{dy} + 5y^2x = 0$ is a linear ODE for $x(y)$.
- (d) False. One solution of $y'^2 + y^2 = 0$ is $y(x) = 0$.
- (e) False. $t^2\frac{d^2t}{dz^2} + 2t\frac{dt}{dz} + 2t = 0$ is an ODE for $t(z)$ and the independent variable, z does not occur explicitly.

3. Solutions of ODEs; Boundary and Initial Value Problems

If

$$y = A_1 e^x + A_2(1 + x),$$

then

$$y' = A_1 e^x + A_2$$

and

$$y'' = A_1 e^x.$$

Inserting this into the ODE $xy'' - (1 + x)y' + y = 0$ gives

$$xA_1 e^x - (1 + x)(A_1 e^x + A_2) + A_1 e^x + A_2(1 + x) = 0.$$

The terms cancel and we obtain $0 = 0$, so $y = A_1 e^x + A_2(1 + x)$ is a solution of the ODE.

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(a) $y(0) = A_1 + A_2 = 1$ and $y(1) = A_1 e + 2A_2 = 1 \implies A_1 = \frac{-1}{e-2}$ and $A_2 = \frac{e-1}{e-2}$.

(b) $y(1) = A_1 e + 2A_2 = 0$ and $y'(2) = A_1 e^2 + A_2 = 0 \implies A_1 = 0$ and $A_2 = 0$.

(c) $y'(1) = A_1 e + A_2 = e$ and $y'(1) = e = y(1) = A_1 e + 2A_2 \implies A_1 = 1$ and $A_2 = 0$

Cases (a) and (b) are boundary value problems and (c) is an initial value problem.