Answer sheet (To be handed in)

Week 7 Coursework Test (2017) | *MATH10222/11222* Calculus and Applications

You Should Try to Answer All Questions

- *Mark with an 'X' only one box corresponding to the correct answer to each of the questions on the question sheet.*
- Important: *negative marking will be used for incorrect answers. You are free not to give an answer if you wish.*
- *The table below indicates the marks awarded for a [r]ight, [w]rong and [n]o answer for each question.*
- *The highest possible total mark is 23 — the lowest mark is zero.*
- *A total mark of 23 counts as 100% for the test.*
- *This document has 9 pages.*

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Name of supervisor: PROF. JOE COOL

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						[r][w][n]
1.	(a) X	(b)				[1][-1][0]
2.	(a)	(b) X	(c)			[2][-1][0]
3.	(a)	(b)	(c) X	(d)		[3][-1][0]
4.	(a) X	(b)				[1][-1][0]
5.	(a)	(b) X				[1][-1][0]
6.	(a)	(b)	(c)	(d) X	(e)	[8][-2][0]
7.	(a)	(b)	(c) X	(d)	(e)	[4][-1][0]
8.	(a)	(b)	(c)	(d) X		[3][-1][0]

At the end of the test please separate this answer sheet from the question sheets and hand it in.

Question sheet (Not to be handed in)

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1. The boundary value problem comprising the linear ODE [1]

$$\frac{d^2y}{dx^2} + Ky = x^2$$

with boundary condition $y(x = 0) = 1$ and $y(x = 1) = 0$ is guaranteed to have a unique solution for all values of the constant K.

(a) $\Longrightarrow$false   NO E & U For BVPs!

(b) true

2. The initial value problem comprising the ODE [2]

$$y'' + \frac{1}{\sin(x)}\, y' - \ y = -1$$

with initial conditions $y(x = 1) = 1$ and $y'(x = 1) = 0$

(a) is guaranteed to have a unique solution for $x \in \mathbb{R}$ because the ODE is linear.

(b) $\Longrightarrow$is guaranteed to have a unique solution for $0 < x < \pi$.

(c) cannot have a solution for $x > \pi$ because the coefficient multiplying y' is singular at $x = \pi$.

3. Which of the following functions is a solution of the ODE [3]

$$\frac{dy}{dx} + \left(\frac{y}{x}\right)^2 = \frac{y}{x}$$

(a) $y(x) = \frac{\ln|x|+C}{x}$

(b) $y(x) = \frac{x}{\ln|x|} + C$

(c) $\Longrightarrow y(x) = \frac{x}{\ln|x|+C}$

(d) $y(x) = \frac{x+C}{\ln|x|}$

where C is an arbitrary constant.

$$\frac{dy}{dx} + \left(\frac{y}{x}\right)^2 = \frac{y}{x} \quad ; \quad z = \frac{y}{x}$$

$$y = xz \; ; \quad \frac{dy}{dx} = z + x\frac{dz}{dx}$$

into ODE:

$$\cancel{z} + x\frac{dz}{dx} + z^2 = \cancel{z} \quad ; \quad x\frac{dz}{dx} = -z^2$$

Separate:

$$\int -z^{-2}\,dz = \int \frac{1}{x}\,dx$$

$$\frac{1}{z} = \frac{x}{y} = \ln|x| + C$$

$$\underline{\underline{y = \frac{x}{\ln|x| + C}}}$$

4. The functions $y_1(x) = x$ and $y_2(x) = a + x$ where a is a constant are linearly independent provided $a \neq 0$. [1]

(a) $\Longrightarrow$true

(b) false

y_1 & y_2 are lin. indep. if $A_1 y_1 + B y_2 = 0 \ \forall x$ requires $A = B = 0$.

So: $Ax + B(x+a) = 0$

$(A+B)x + Ba = 0 \quad \forall x$

choose $x=0$: $B = 0$ (unless $a = 0$)

$x = 1$: $A = 0$

5. The *general* solution to the homogeneous ODE [1]

$$\sin(x+y)\, y'' + x^2\, y' = 0$$

is given by $y(x) = Ay_1(x) + By_2(x)$ where A and B are arbitrary constants and $y_1(x)$ and $y_2(x)$ are *any* two nonzero, linearly independent solutions of the ODE.

(a) true

(b) $\Longrightarrow$false

ODE is nonlinear.

6. The general solution of the ODE [8]

$$y'' - y = \exp(x)$$

is given by

(a) $y(x) = A\exp(x) + B\exp(-x) + 2x\exp(x)$

(b) $y(x) = A\exp(x) + B\exp(-x) + 2\exp(x)$

(c) $y(x) = A\sin(x) + B\cos(x) + 2x\exp(x)$

(d) $\Longrightarrow y(x) = A\exp(x) + B\exp(-x) + \frac{1}{2}x\exp(x)$

(e) $y(x) = A\sin(x) + B\cos(x) + \frac{1}{2}x\exp(x)$

where A and B are arbitrary constants.

$y'' - y = \exp(x)$

$y = y_H + y_P$

$y_H'' - y_H = 0$; $y_H \sim e^{\lambda x}$; $\lambda^2 - 1 = 0$

$\Rightarrow \lambda_{12} = \pm 1 \Rightarrow y_H = A\exp(x) + B\exp(-x)$

$y_P = C\exp(x)$ won't work $\Rightarrow$

$y_P = Cx\exp(x)$; $y_P' = C(x\exp(x) + \exp(x))$

$y_P'' = C(x\exp(x) + 2\exp(x))$

into ODE:

$C(\cancel{x\exp(x)} + 2\exp(x) - \cancel{x\exp(x)}) \overset{!}{=} \exp(x)$

$\Rightarrow C = \frac{1}{2}$

So

$y = \frac{1}{2}x\exp(x) + A\exp(x) + B\exp(-x)$

7. The general solution of the ODE [4]

$$\frac{dy}{dx} + 2xy = 2x$$

is given by

(a) $y(x) = 1 + C\exp(x^2)$

(b) $y(x) = C + \exp(x^2)$

(c) $\Longrightarrow y(x) = 1 + C\exp(-x^2)$

(d) $y(x) = C + \exp(-x^2)$

(e) $y(x) = C(1 + \exp(-x^2))$

where C is an arbitrary constant.

$$\frac{dy}{dx} + \underbrace{2x}_{p(x)}\, y = \underbrace{2x}_{q(x)} \quad \text{lin. ODE}$$

Integr. factor $\mu(x) = \exp\left(\int 2x\, dx\right) = \exp(x^2)$

$$y(x)\mu(x) = \int 2x \exp(x^2)\, dx = \int \exp(z)\, dz$$

$$z = x^2; \quad dx = \frac{dz}{2x}$$

$$= \exp(x^2) + C$$

$$\underline{\underline{y(x)}} = \frac{\exp(x^2) + C}{\exp(x^2)} = \underline{\underline{1 + C\exp(-x^2)}}$$

8. The solution of the initial value problem [3]

$$\frac{dy}{dx} = \frac{x}{3y^2}\,\exp(-y^3), \qquad y(0) = 1$$

is given by

(a) $y(x) = \left[\ln(\frac{1}{3}x^3 - e)\right]^{1/2}$

(b) $y(x) = \left[\ln(x^3 + e)\right]^{1/3}$

(c) $y(x) = \left[\ln(\frac{1}{2}x^2 - e)\right]^{1/2}$

(d) $\Longrightarrow y(x) = \left[\ln(\frac{1}{2}x^2 + e)\right]^{1/3}$

$$\frac{dy}{dx} = \frac{x}{3y^2}\exp(-y^3) \qquad y(0) = 1$$

Separate:

$$\int 3y^2 \exp(+y^3)\,dy = \int x\,dx$$

$$z = y^3;\ dy = \frac{dz}{3y^2}$$

$$\int \exp(z)\,dz = \int x\,dx$$

$$\exp(\eta^3)\Big|_{\eta=1}^{y} = \frac{1}{2}\xi^2\Big|_{\xi=0}^{x}$$

$$\exp(y^3) - e = \frac{1}{2}x^2$$

$$\underline{y = \left[\ln(\tfrac{1}{2}x^2 + e)\right]^{1/3}}$$