

MATH10222: EXAMPLE SHEET¹ I*Questions for supervision classes*

Hand in the solutions to *all* questions on this sheet for your supervision classes. If you have a new supervisor and the procedure for handing in solutions has not yet been established, attempt all questions and raise any problems that you encounter with your supervisor.

1. Classifying ODEs

Classify the following ODEs: Are they linear or nonlinear, autonomous or non-autonomous, and what is their order?

- (a) $u'(x) + u^2(x) = \cos x$.
- (b) $\frac{d^4u(t)}{dt^4} + (2 + \cos(t)) u(t) = 3$.
- (c) $\ddot{\theta} + \Omega^2 \sin \theta = 0$, where the constant Ω is given.

2. Properties of ODEs

True or false? Briefly justify every answer. (Be careful – some of these are tricky!)

- (a) $\left(\frac{d\phi}{ds}\right)^2 = 2s\phi$ is a second order ODE.
- (b) $u''(x) + u'(x - 1) = 1$ is a second-order ODE.
- (c) $\frac{dx}{dy} + 5xy^2 = 0$ is not a linear ODE.
- (d) The ODE $y'^2 + y^2 = 0$ has no real solutions.
- (e) $t^2 \frac{d^2t}{dz^2} + 2t \frac{dt}{dz} + 2t = 0$ is not an autonomous ODE.

3. Solutions of ODEs; Boundary and Initial Value Problems

Confirm that $y = A_1 e^x + A_2(1 + x)$ is a solution of the ODE $xy'' - (1 + x)y' + y = 0$.

Find the values of the constants A_1 and A_2 that are determined by each of the following pairs of constraints. For each case, state whether the combination of the ODE and the constraint constitutes a boundary value problem or an initial value problem.

- (a) $y(0) = 1$ and $y(1) = 1$.
- (b) $y(1) = 0$ and $y'(2) = 0$.
- (c) $y'(1) = e$ and $y'(1) = y(1)$.

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