My research interests are primarily in noncommutative algebra although there are substantial overlaps between this and neigbouring subjects like algebraic geometry and Lie theory

A particular interest is in "noncommutative algebraic geometry." There are almost as many definitions of what this means as there are people working in it, but for me it has two complimentary components. The first is to use the techniques and intuition from (especially projective) algebraic geometry to understand the structure of important noncommutative algebras, while the second is to build up a theory of noncommutative geometry itself. Curiously, some of the most substantial applications of this principle occur with projective rather than affine geometry. This is perhaps due to the fact that the techniques of affine geometry tend to rely on techniques like localisation that are rarely availably in the noncommutative universe, whereas the more global and categorical approaches to projective geometry can and have been generalized. We have used this, for example, to classify all noncommutative graded rings of quadratic growth (sometimes called noncommutative curves) in terms of commutative curves. The next step, that of classifying noncommutative surfaces, or graded algebras with cubic growth. A solution of this problem is not in sight, but this has lead to the construction of some fascinating algebras and techniques that can be widely applied.

Another area of interest is in rings of differential operators. The classic example, here, is the Weyl algebra consisting of all differential operators with polynomial coefficients or, equivalently, the ring of all differential operators that map the polynomial ring in n variables into itself. This is a very well-behaved ring, although there are still many unresolved questions about it. However, if one takes a nonregular commutative ring then much less is known about the corresponding ring of differential operators. In many cases these rings can be shown to be factor rings of enveloping algebras of simple Lie algebras, and this can be very useful in understanding the representation theory of those Lie algebras.

A recent interest of mine is in the so-called Cherednik algebras and their spherical subalgebras. These behave rather like enveloping algebras of Lie algebras—and are in fact closely related to the rings of differential operators mentioned in the last paragraph. But they are also related to important geometric constructs like Hilbert schemes of points.