Supplement to "An examination of the SEP Candidate Analogical Inference Rule within Pure Inductive Logic"

The following is intended as a supplement to [1] and adopts the same notation etc.. In particular we continue to assume that the probability function w on L satisfies Ex + Px + SN.

Theorem 1. *Let L be the unary language with a single predicate symbol R. Then for a probability function w on L not of the form $\lambda c_0^L + (1-\lambda)w'$ with $0 < \lambda < 1$, $w' \neq c_0^L$ and $\phi(a_1), \psi(a_1) \in SL$ mentioning only the constant symbol a_1,*

$$w(\phi(a_2) \,|\, \phi(a_1) \wedge \psi(a_1) \wedge \psi(a_2)) \geq w(\phi(a_2) \,|\, \phi(a_1)). \tag{1}$$

Conversely if w is of that form then there are such $\phi(a_1), \psi(a_1)$ for which (1) fails.

Proof. From the well known representation theorem for unary languages there are subsets S_ϕ, S_ψ of

$$\{\forall x\, R(x),\ \forall x\, \neg R(x),\ R(a_1) \wedge \exists x\, \neg R(x),\ \neg R(a_1) \wedge \exists x\, R(x)\}$$

such that

$$\phi(a_1) \equiv \bigvee S_\phi, \qquad \psi(a_1) \equiv \bigvee S_\psi.$$

The proof now consists of considering lots of cases.

Firstly (1) immediately holds if $S_\psi \subseteq S_\phi$ or $S_\phi \cap S_\psi = \emptyset$. It also directly holds if $S_\psi \cap S_\phi \subseteq \{\forall x\, R(x),\ \forall x\, \neg R(x)\}$ since in this case

$$\phi(a_1) \wedge \psi(a_1) \equiv \phi(a_1) \wedge \psi(a_1) \wedge \psi(a_2) \equiv \phi(a_2) \wedge \phi(a_1) \wedge \psi(a_1) \wedge \psi(a_2)$$

Discounting these cases then we are left with

1. $\phi(a_1) \equiv (R(a_1) \wedge \exists x\, \neg R(x)) \vee \bigvee T_1$
   $\psi(a_1) \equiv (R(a_1) \wedge \exists x\, \neg R(x)) \vee \bigvee T_2$

2. $\phi(a_1) \equiv (\neg R(a_1) \wedge \exists x\, R(x)) \vee \bigvee T_1$
   $\psi(a_1) \equiv (\neg R(a_1) \wedge \exists x\, R(x)) \vee \bigvee T_2$

3. $\phi(a_1) \equiv (R(a_1) \wedge \exists x\, \neg R(x)) \vee (\neg R(a_1) \wedge \exists x\, R(x)) \vee \bigvee T_1$
   $\psi(a_1) \equiv (R(a_1) \wedge \exists x\, \neg R(x)) \vee (\neg R(a_1) \wedge \exists x\, R(x)) \vee \bigvee T_2$

4. $\phi(a_1) \equiv (R(a_1) \wedge \exists x\, \neg R(x)) \vee \bigvee T_1$
   $\psi(a_1) \equiv (R(a_1) \wedge \exists x\, \neg R(x)) \vee (\neg R(a_1) \wedge \exists x\, R(x)) \vee \bigvee T_2$

5. $\phi(a_1) \equiv (\neg R(a_1) \wedge \exists x\, R(x)) \vee \bigvee T_1$
   $\psi(a_1) \equiv (R(a_1) \wedge \exists x\, \neg R(x)) \vee (\neg R(a_1) \wedge \exists x\, R(x)) \vee \bigvee T_2$

6. $\phi(a_1) \equiv (R(a_1) \wedge \exists x\, \neg R(x)) \vee (\neg R(a_1) \wedge \exists x\, R(x)) \vee \bigvee T_1$
   $\psi(a_1) \equiv (R(a_1) \wedge \exists x\, \neg R(x)) \vee \bigvee T_2$

7. $\phi(a_1) \equiv (R(a_1) \wedge \exists x\, \neg R(x)) \vee (\neg R(a_1) \wedge \exists x\, R(x)) \vee \bigvee T_1$
   $\psi(a_1) \equiv (\neg R(a_1) \wedge \exists x\, R(x)) \vee \bigvee T_2$

for some $T_1, T_2 \subseteq \{\forall x\, R(x),\, \forall x\, \neg R(x)\}$.

In case 1. (1) holds because

$$\phi(a_1) \wedge \psi(a_1) \wedge \psi(a_2) \models (R(a_2) \wedge \exists x\, \neg R(x)) \vee \bigvee T_1 \models \phi(a_2)$$

and similarly in cases 2,3,6,7. In case 5., $\phi(a_1) \wedge \psi(a_1) \wedge \psi(a_2)$ is logically equivalent to

$$\big((\neg R(a_1) \wedge \exists x\, R(x)) \wedge \big((R(a_2) \wedge \exists x\, \neg R(x)) \vee (\neg R(a_2) \wedge \exists x\, R(x))\big)\big) \vee \bigvee (T_1 \cap T_2),$$

equivalently

$$(\neg R(a_1) \wedge \neg R(a_2) \wedge \exists x\, R(x)) \vee (\neg R(a_1) \wedge R(a_2)) \vee \bigvee (T_1 \cap T_2).$$

Also in this case $\phi(a_2) \wedge \phi(a_1) \wedge \psi(a_1) \wedge \psi(a_2)$ is logically equivalent to

$$(\neg R(a_1) \wedge \neg R(a_2) \wedge \exists x\, R(x)) \vee \bigvee (T_1 \cap T_2)$$

so $w(\phi(a_2) \,|\, \phi(a_1) \wedge \psi(a_1) \wedge \psi(a_2))$ equals

$$\frac{w(\neg R(a_1) \wedge \neg R(a_2) \wedge \exists x\, R(x)) + w\big(\bigvee (T_1 \cap T_2)\big)}{w(\neg R(a_1) \wedge R(a_2)) + w(\neg R(a_1) \wedge \neg R(a_2) \wedge \exists x\, R(x)) + w\big(\bigvee (T_1 \cap T_2)\big)} \tag{2}$$

whilst $w(\phi(a_2) \,|\, \phi(a_1))$ equals

$$\frac{w(\neg R(a_1) \wedge \neg R(a_2) \wedge \exists x\, R(x)) + w\big(\bigvee T_1\big)}{w(\neg R(a_1) \wedge \exists x\, R(x)) + w\big(\bigvee T_1\big)} =$$

$$\frac{w(\neg R(a_1) \wedge \neg R(a_2) \wedge \exists x\, R(x)) + w\big(\bigvee T_1\big)}{w(\neg R(a_1) \wedge R(a_2)) + w(\neg R(a_1) \wedge \neg R(a_2) \wedge \exists x\, R(x)) + w\big(\bigvee T_1\big)}. \tag{3}$$

If w is not of the form $\lambda c_0^L + (1-\lambda)w'$ with $0 < \lambda < 1$, $w' \neq c_0^L$ then either

$$w(\neg R(a_1) \wedge R(a_2)) = w(\neg R(a_1) \wedge \neg R(a_2) \wedge \exists x\, R(x)) = 0$$

or

$$w(\forall x\, R(x)) = w(\forall x\, \neg R(x)) = 0$$

in which case (3)$\leq$ (2) holds. On the other hand if w is of that form then we can arrange (3)>(2) by taking

$$T_1 = \{\forall x\, R(x),\ \forall x\, \neg R(x)\}, \quad T_2 = \{\forall x\, \neg R(x)\}.$$

Finally case 4. follows analogously to case 5.. □

References

[1] Howarth, E., Paris, J.B. & Vencovská, A., An examination of the SEP Candidate Analogical Inference Rule within Pure Inductive Logic. To be submitted.