A Note on Carnap's Continuum and the Weak State Description Analogy Principle

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Abstract

Under the assumption of Atom Exchangeability and Regularity we give a characterization of the probability functions in Carnap's Continuum of Inductive Methods in terms of the Weak State Description Analogy Principle.

Key words: State Description Analogy Principle, Carnap's Continuum, Pure Inductive Logic, Analogy, Rationality.

The Characterization

We shall assume the context and notation of Unary Pure Inductive Logic as set out in [2]. Throughout w will be a probability function on the set SL of sentences of the unary language L with $q \geq 2$ predicate symbols which satisfies Ex.

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For atoms

$$\alpha_j = \bigwedge_{i=1}^{q} R_i^{\epsilon_i}(x), \quad \alpha_k = \bigwedge_{i=1}^{q} R_i^{\delta_i}(x)$$

set

$$\Delta(\alpha_r, \alpha_m) = \{ i \,|\, \epsilon_i = \delta_i \}.$$

The Weak State Description Analogy Principle, WSDAP

For atoms $\alpha_r, \alpha_m, \alpha_k$, if $\Delta(\alpha_r, \alpha_m) \subseteq \Delta(\alpha_r, \alpha_k)$ then

$$w(\alpha_r(a_{n+2}) \,|\, \alpha_k(a_{n+1}) \wedge \bigwedge_{i=1}^{n} \alpha_{h_i}(a_i)) \leq w(\alpha_r(a_{n+2}) \,|\, \alpha_m(a_{n+1}) \wedge \bigwedge_{i=1}^{n} \alpha_{h_i}(a_i)). \quad (1)$$

Theorem 1. *Let w satisfy Atom Exchangeability and Regularity. Then w satisfies WSDAP just if $w = c_\lambda^L$ for some $0 < \lambda \leq \infty$.*

Proof. First suppose that w satisfies WSDAP. Let r, m, k be distinct and such that

$$\Delta(\alpha_r, \alpha_m) \subseteq \Delta(\alpha_r, \alpha_k).$$

By WSDAP we must have

$$w(\alpha_r(a_{n+2}) \,|\, \alpha_m(a_{n+1}) \wedge \bigwedge_{i=1}^{n} \alpha_{h_i}(a_i)) \leq w(\alpha_r(a_{n+2}) \,|\, \alpha_k(a_{n+1}) \wedge \bigwedge_{i=1}^{n} \alpha_{h_i}(a_i)). \quad (2)$$

But we can (for $q \geq 2$) produce a permutation σ of $\{1, 2, \ldots, 2^q\}$ which leaves r fixed and is such that

$$\Delta(\alpha_r, \alpha_{\sigma(m)}) \supset \Delta(\alpha_r, \alpha_{\sigma(k)})$$

whilst by using Atom Exchangeability, Ax, on (2) we also have that

$$w(\alpha_r(a_{n+2}) \,|\, \alpha_{\sigma(m)}(a_{n+1}) \wedge \bigwedge_{i=1}^{n} \alpha_{\sigma(h_i)}(a_i)) \leq w(\alpha_r(a_{n+2}) \,|\, \alpha_{\sigma(k)}(a_{n+1}) \wedge \bigwedge_{i=1}^{n} \alpha_{\sigma(h_i)}(a_i)).$$

This is only consistent with WSDAP if we actually have equality in (2).

Indeed by the transitivity of equality we must also have equality in (2) for *any* α_m, α_k not equal to α_r. From this it follows that for state descriptions

$$\Theta = \bigwedge_{i=n+1}^{m} \alpha_{h_i}(a_i), \quad \Phi = \bigwedge_{i=n+1}^{m} \alpha_{g_i}(a_i)$$

where r does not appear amongst the h_i or g_i, we must have that

$$w(\alpha_r(a_1) \,|\, \bigwedge_{i=2}^{n} \alpha_r(a_i) \wedge \Theta) = w(\alpha_r(a_1) \,|\, \bigwedge_{i=2}^{n} \alpha_r(a_i) \wedge \Phi). \tag{3}$$

For if this failed then, by changing one atom at a time and using Ex, it would have to fail in a case when just $h_{n+1} \neq g_{n+1}$ and $h_j = g_j$ for $n+2 \leq j \leq m$, which would contradict equality holding in (2). But, under Ax, (3) is just Johnson's Sufficientness Postulate whose only solutions w for $q \geq 2$ and w regular are Carnap's c_λ^L for $0 < \lambda \leq \infty$.

In the other direction it is straightforward to check by direct calculation that these c_λ^L satisfy WSDAP. □

Unfortunately, as pointed out in [3], in the case of the c_λ^L $(\lambda > 0)$ WSDAP as above holds with equality in (2) so it can hardly be said to capture more than a faint hint of analogy. From other work in this area, see for example [1], it is clear that this collapse is very much a consequence of assuming Ax, much more structure remains if instead we only assume Px+SN.

References

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