1C1/0C1 COURSEWORK 2, 2011, SOLUTIONS[1]

1. (i) By inspection $x^2+3x-18=(x+6)(x-3)$, so the solutions are $-6, 3$. Alternatively, using the formula,

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(3)\pm\sqrt{3^2-4\times(-18)}}{2}=\frac{-3\pm\sqrt{81}}{2}=\frac{-3\pm 9}{2}=-6,3.$$

(ii) Using the formula the solutions are

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{1\pm\sqrt{1-4\times(2)(-2)}}{4}=\frac{1\pm\sqrt{17}}{4}.$$

[This answer is perfectly correct as it stands, do not use your calculator to produce instead an answer which is only correct to a few decimal places!]

(iii)

$$\begin{aligned}\frac{x-5}{3}+\frac{x+1}{5}=-2 &\iff 5(x-5)+3(x+1)=(-2)\times 15=-30 \text{ (multiply both sides by 15)}\\ &\iff 5x-25+3x+3=-30\\ &\iff 8x=-8\\ &\iff x=-1.\end{aligned}$$

(iv)

$$\begin{aligned}\frac{2}{x+3}+\frac{1}{12-2x}=\frac{1}{2} &\iff 2.2(12-2x)+2(x+3)=(x+3)(12-2x) \text{ (multiply both sides by } 2(x+3)(12-2x))\\ &\iff 48-8x+2x+6=12x-2x^2+36-6x\\ &\iff 2x^2-12x+18=0\\ &\iff 2(x-3)(x-3)=0\\ &\iff x=3.\end{aligned}$$

(v)

$$\begin{aligned}\frac{x-1}{3-x}=\frac{x}{x+3} &\iff (x-1)(x+3)=x(3-x) \text{ (multiply both sides by } (3-x)(x+3))\\ &\iff x^2+2x-3=3x-x^2\\ &\iff 2x^2-x-3=0\\ &\iff x=\frac{1\pm\sqrt{(-1)^2-4(2)(-3)}}{2\times 2} \text{ (by the formula)}\\ &\iff x=\frac{1\pm 5}{4}\\ &\iff x=3/2,-1.\end{aligned}$$

(vi) Put $y=3^x$. Then since $9^x=(3^2)^x=3^{2x}=(3^x)^2$ the equation becomes $y^2-10y+9=0$. By inspection (or use the formula), this factorizes to
$(y-9)(y-1)=0$, so $y=9$ or $y=1$. Hence $3^x=9$ or $3^x=1$, giving the solutions $x=2,0$.

2. (i) $1/3=\sin(A)=a/4$ so $a=4/3$.

(ii) $\sin^2(A)+\cos^2(A)=1$ so $\cos^2(A)=1-(1/3)^2=8/9$ and $\cos(A)=\sqrt{8/9}=2\sqrt{2}/3$ (positive root since $0\le A\le\pi/2$).

(iii) $\cos(A)=b/4$ so $b=4\times(2\sqrt{2}/3)=8\sqrt{2}/3$.

(iv) $\cos(2A)=2\cos^2(A)-1=2\left(\frac{2\sqrt{2}}{3}\right)^2-1=2\times\left(\frac{8}{9}\right)-1=\frac{16}{9}-1=\frac{7}{9}$.

[1] These solutions are intended to help students who are struggling and are therefore very detailed. In an exam you could safely skip some of these steps if you wished.

3.

(i) If $(-3,-4)$ was on the line $y = 3x + 2$ then this equation would hold if we substituted -3 for x and -4 for y. But when we make this substitution we get $-4 = 3 \times (-3) + 2 = -7$ which does not hold. Hence $(-3,-4)$ is not on the line $\mathcal{E}$.

(ii) At the point (x, y) at which the lines $y = 3x + 2$ and $y = -2x + 7$ cross both these equations must hold. Hence $3x+2 = -2x+7$, i.e. $x = 1$ and substituting this value for x gives $y = 3\times 1+2 = 5$. So the point of intersection is $(1, 5)$.

(iii) $\mathcal{E}$ has gradient 3 so the required parallel line $y = mx + c$ must have gradient 3, i.e. $m = 3$, and pass through $(3, 3)$, so $3 = m \times 3 + c$. Substituting 3 for m gives $c = -6$ so the required line is $y = 3x - 6$.

(iv) Since the gradient of $\mathcal{E}$ is 3 the slope m of the normal to $\mathcal{E}$ through the point $(3, 3)$ must satisfy $3m = -1$, i.e. $m = -1/3$. So the line is of the form $y = -x/3 + c$ and since it passes through $(3, 3)$, $3 = -3/3 + c$, so $c = 4$. Hence the required line is $y = -x/3 + 4$.

(v) If the line $y = mx+c$ goes through the points $(-1, 2)$ and $(3, 6)$ we must have $2 = -m+c$ and $6 = 3m+c$. Subtracting the first of these from the second gives $4 = 4m$ so $m = 1$ and then substituting this value of m into the first equation gives $2 = -1 + c$ so $c = 3$. Thus the required line is $y = x + 3$.

FEEDBACK

Most students did quite well. Nevertheless if you scored 9 or less you should certainly study these solutions and make sure you don't make the same mistakes in the exam – where you will have far less time time available.

Most of the marks which students lost were through pure carelessness and could have been avoided by simply checking the working. Given that you had as much time as you wanted for this test this is really inexcusable. Make sure in the exam that you do check your working if you have time over at the end.

In questions like 1 where you had to solve an equation, if the answer comes out as a nice round figure you can check it by substituting back into the original equation. This is an easy way to confirm that you're right which, it seems, many students did not avail themselves of. Similarly in question 3(v) you can check that your line goes through the two points in question by making sure these points actually lie on the line (as in question 3(i) which just about everyone got right).

In marking these questions I allowed answers which were not necessarily in their simplest form, for example $\sqrt{8/9}$ in 2(ii) rather than $2\sqrt{2}/3$. You should however endeavor to give simplify your answers as far as possible since this will often be asked for explicitly in the question (as on the Coursework 1).

As with Coursework 1 some students lost marks by simply not answering the question. For example giving $b/4$ for $\cos(a)$ in 2(ii) or factorizing $x^2 + 3x - 18$ in 1(i) as $(x - 3)(x + 6)$ and leaving it at that without actually stating what the solutions to the original equation were.

In 2(ii) and 2(iv) some students came up with values of $\cos(A)$ ($\cos(2A)$) which were greater than 1 or less than -1. It should be clear to you from the graph of $y = \cos(x)$, or directly from its definition, that values of $\cos(x)$ are always between -1 and 1 so if you get a value value outside of this range it should set alarm bells ringing! Similarly for $\sin(x)$.

The rubric said that calculators were not permitted. Answers which had clearly been obtained with the aid of a calculator received no marks.

A common mistake was taking the formula for the roots of a quadratic to be

$$-b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

rather than the correct

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

A second common mistake was taking the gradient of a line normal to the line $y = mx + c$ to be $-m$ rather than the correct $-1/m$. Make sure you don't make these mistakes in the exam.