1. 
   (1)(i) \((x - 3)(x^2 + 5) = x(x^2 + 5) - 3(x^2 + 5) = x^3 + 5x - 3x^2 - 15\)
   (1)(ii) \((a - b - 2)(a + b - 1) = a(a + b - 1) - b(a + b - 1) - 2(a + b - 1)\)
   \[= a^2 + ab - a - ba - b^2 + b - 2a - 2b + 2 = a^2 - b^2 - 3a - b + 2\]
   (1)(iii) \((1 - x)(2 - (x + 3)) = (1 - x)(-1 - x) = -1 - x + x + x^2 = x^2 - 1\)
   (1)(iv) \(x(x - 1)(1 - 3x) = x(x - 3x^2 - 1 + 3x) = x(-3x^2 + 4x - 1) = -3x^3 + 4x^2 - x.\)
   (2) In 1(iv) the term in \(x^3\) is \(-3x^3\), the coefficient of \(x\) is \(-1\) and the constant term is 0.

3. 
   (i) \(\frac{x^4}{x^7} = x^{4-7} = x^{-3}\)  \(\text{ (ii) } x^2 \sqrt{x} = x^{2+1/2} = x^{7/3}\)  \(\text{ (iii) } (x^4)^{1/6} = x^{4 \times 1/6} = x^{4/6} = x^{2/3}\)

2. (1) \(x^2 + 3x - 10 = (x - 2)(x + 5)\) so \(x^2 + 3x - 10 = 0 \iff x = 2, -5.\)
   Or use the formula to give solutions
   \[\frac{-3 \pm \sqrt{3^2 - 4.(-10)}}{2} = \frac{-3 \pm \sqrt{49}}{2} = \frac{-3 \pm 7}{2} = 2, -5.\]
   (2) \(3x^2 - x - 5 = x^2 - 2x - 3 \iff 2x^2 + x - 2 = 0 \iff x = \frac{-1 \pm \sqrt{1^2 - 4.2.(-2)}}{4} \iff x = \frac{-1 \pm \sqrt{17}}{4}.\)
   (3) \(\frac{x + 1}{x - 3} = \frac{x - 1}{3} \iff 3(x + 1) = (x - 3)(x - 1) \iff 3x + 3 = x^2 - 4x + 3 \iff x^2 - 7x = 0 \iff x(x - 7) = 0 \iff x = 0, 7.\)
\[
\frac{1}{4-x} + \frac{1}{x} = \frac{-2}{x+8} \quad \iff \quad x(x+8) + (x+8)(4-x) = -2x(4-x)
\]
\[
\iff x^2 + 8x - x^2 - 4x + 32 = 2x^2 - 8x
\]
\[
\iff 2x^2 - 12x - 32 = 0
\]
\[
\iff x^2 - 6x - 16 = 0
\]
\[
\iff (x+2)(x-8) = 0
\]
\[
\iff x = -2, 8 \quad \text{(or by using the formula)}
\]

(5) Put \( y = x^2 \), so the equation becomes
\[
y^2 - 3y + 2 = 0 \quad \iff \quad (y-2)(y-1) = 0 \quad \iff \quad y = 1, 2
\]
Hence \( x^2 = 1, 2 \) so \( x = 1, -1, \sqrt{2}, -\sqrt{2} \).

3. (i) \( 25^x = 5 \quad \iff \quad (5^2)^x = 5 \quad \iff \quad 5^{2x} = 5 \quad \iff \quad 2x = 1 \quad \iff \quad x = 1/2 \).

(ii) \( \log_5 \left( \frac{4}{x-1} \right) = -1 \quad \iff \quad \left( \frac{1}{x-1} \right) = 5^{-1} \quad \iff \quad \frac{4}{x-1} = \frac{1}{5} \)
\[
\iff x - 1 = 20 \quad \iff \quad x = 21
\]

(iii) \( \log_3 (9^{x+2}) = 3x \quad \iff \quad (x+2) \log_3 (9) = 3x \)
\[
\iff 2(x+2) = 3x \quad \iff \quad x = 4.
\]

(iv) \( x \log_2 (x) = \log_3 (x) \quad \iff \quad \frac{x \log_3 (x)}{\log_3 (2)} = \log_3 (x) \quad \iff \quad x = \log_3 (2) \) using the change of base formula \( \log_a (x) = \frac{\log_b (x)}{\log_b (a)} \).

or \( x \log_2 (x) = \log_3 (x) \quad \iff \quad x \log_2 (x) = \frac{\log_2 (x)}{\log_2 (3)} \quad \iff \quad x = \frac{1}{\log_2 (3)} \)

(v) \( \log_x (x^2 - 5x + 9) = 1 \quad \iff \quad x^2 - 5x + 9 = x^1 \quad \iff \quad x^2 - 6x + 9 = 0 \quad \iff \quad (x - 3)(x - 3) = 0 \quad \iff \quad x = 3.\)

4. (1) If \( y = mx + c \) passes through both \((-1,4)\) and \((1,8)\) then
\[
4 = -m + c \quad \text{and} \quad 8 = m + c.
\]
Subtracting the first equation from the second gives \( 4 = 2m \) so \( m = 2 \) and with the second equation this gives \( 8 = (2)(1) + c \) so \( c = 6 \) and the equation of the line \( C \) is \( y = 2x + 6 \).

(2) Substituting \( x = -2, y = 2 \) into \( y = 2x + 6 \) gives \( 2 = (2)(-2) + 6 \) which is true so this point lies on the line \( C \).

(3) If \( y = 0 \) then \( 0 = 2x + 6 \) and \( x = -3 \) so \( C \) crosses the \( x \)-axis at the point \( A = (-3,0) \).

(4) The distance from \((-3,0)\) to \((1,8)\) is
\[
\sqrt{((-3-1)^2 + (0-8)^2} = \sqrt{16+64} = \sqrt{80} = \sqrt{16\sqrt{5}} = 4\sqrt{5}.
\]

(5) Considering the right angled triangle formed by the points \( A = (-3,0) \), \((1,8)\) and \((1,0)\) (notice that \((-3,0)\) and \((1,8)\) lie on the line \( C \) and \((-3,0)\) and \((1,0)\) lie on the \( x \)-axis) we
see that the cosine of the angle \( C \) makes with the \( x \)-axis is the distance from \((-3,0)\) to \((1,0)\), divided by the distance from \((-3,0)\) to \((1,8)\), i.e. \(4/\sqrt{5} = 1/\sqrt{5}\).

(6) If the lines \( y = 2x + 6 \) and \( y = 14 - 2x \) intersect at \( (x,y) \) then

\[
2x + 6 = y = 14 - 2x
\]

so \( 2x + 2x = 14 - 6 \), i.e. \( x = 2 \) and \( y = (2)(2) + 6 = 10 \). Hence the two lines intersect at the point \((2,10)\).

5. (1) At the point of intersection we must have \(3x + 1 - x^2 = y = 5 - x\). Hence \(0 = x^2 - 4x + 4 = (x - 2)(x - 2)\), so \( x = 2 \). For this value of \( x \), \( y = 5 - 2 = 3 \) so the required point is \((2,3)\).

(2) At \( A \) the slope of \( E \) is -1 and the slope of \( C \) is \( d/dx(3x + 1 - x^2) = 3 - 2x \) evaluated at \( x = 2 \), i.e. -1 again. So \( C \) and \( E \) are parallel at \((2,3)\) and \( E \) must be the tangent to \( C \) at this point.

(3) Let the normal be \( y = mx + c \). Since it is normal to \( E \) its slope \( m \) must satisfy \( m(-1) = -1 \) so \( m = 1 \). Also since this line goes through the point \((2,3)\), \( 3 = 2m + c \). Hence \( c = 1 \) and the normal is \( y = x + 1 \).

(4) The normal intersects \( C \) when \( 3x + 1 - x^2 = y = x + 1 \). Hence \( 0 = x^2 - 2x = x(x - 2) \). The solution \( x = 2 \) gives the point \( A \) so the other point on intersection must be when \( x = 0 \), and \( y = 0 + 1 = 1 \), i.e. the point \((0,1)\).

(5) The slope of \( C \) at \( x = 0 \) is given by \( d/dx(3x + 1 - x^2) = 3 - 2x \) evaluated at \( x = 0 \), i.e. 3. Hence the required tangent \( y = mx + c \) to \( C \) at \((0,1)\) satisfies \( m = 3 \), \( 1 = m(0) + c \), so the tangent here is \( y = 3x + 1 \).

6. (a)

\[
\begin{align*}
\cos(A) &= b/7 \text{ so } b = 7\cos(A) = 14/3. \\
\sin^2(A) + \cos^2(A) &= 1 \text{ so } \sin^2(A) = 1 - 4/9 = 5/9 \text{ and } \sin(A) = \sqrt{5/9} = \sqrt{5}/3. \\
a/7 &= \sin(A) \text{ so } a = 7\sin(A) = 7\sqrt{5}/3. \\
\cos(B) &= a/7 = \sin(A) = \sqrt{5}/3. \\
\sin(B) &= b/7 = \cos(A) = 2/3.
\end{align*}
\]

(b) \( 2\cos(A)\cos(B) = \cos(A + B) + \cos(A - B) \quad \star \)

Substituting \( A = B = \pi/8 \) in \( \star \) gives

\[
2\cos^2(\pi/8) = \cos(\pi/4) + \cos(0) = 1/\sqrt{2} + 1
\]

so

\[
\cos(\pi/8) = \sqrt{\left(\frac{1 + 1/\sqrt{2}}{2}\right)} = \sqrt{\left(\frac{1 + \sqrt{2}}{2\sqrt{2}}\right)}.
\]
Substituting $A = \pi/4$, $B = \pi/8$ into $\star$ gives

$$2 \cos(\pi/4) \cos(\pi/8) = \cos(3\pi/8) + \cos(\pi/8)$$

so

$$\cos(3\pi/8) = ((2/\sqrt{2}) - 1) \cos(\pi/8) = (\sqrt{2} - 1) \cos(\pi/8).$$

7. (1)

(i) $d/dx(3x^3 - 3) = 9x^2$

(ii) $d/dx(x^{1/3}) = \frac{x^{-1+1/3}}{3} = \frac{x^{-2/3}}{3}$

(iii) $d/dx(e^{2x-1}) = 2e^{2x-1}$

(2) For $f(x) = 2x^3 - 3x^2 - 12x + 20$, $df/dx = 6x^2 - 6x - 12$ and $d^2f/dx^2 = 12x - 6$. Hence the stationary points are when

$$df/dx = 3x^2 - 6x - 12 = 6(x - 2)(x + 1) = 0,$$

i.e. $x = 2, -1$. When $x = 2$ $d^2f/dx^2 = 12(2) - 6 = 18 > 0$ so this is a (local) minimum. When $x = -1$, $d^2f/dx^2 = 12(-1) - 6 = -18 < 0$ so this is a (local) maximum point.

The graph of this function looks as below.
In other words it goes up from the left to reach a local maximum at \( x = -1 \), when in fact 
\[ f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 20 = 27. \] 
It then goes down to a local minimum at \( x = 2 \) (when \( f(1) = 2(2)^3 - 3(2)^2 - 12(2) + 20 = 0 \) ) and thereafter moves up to the right.] Since the line \( y = -15 \) lies below the curve at the local minimum this line only cuts the curve (to the left of \( x = -1 \)) in one place so \( 2x^3 - 3x^2 - 12x + 20 = -15 \) has only one solution.

8.

1. \[ \frac{d}{dx} (x + 1)^2 \sin(x) = \sin(x) \frac{d}{dx} (x + 1)^2 + (x + 1)^2 \frac{d}{dx} \sin(x) \]
   \[ = 2(x + 1) \sin(x) + (x + 1)^2 \cos(x) \]

2. \[ \frac{d}{dx} (\cos^2(x)) = \frac{d}{dx} (\cos(x)) \cdot \cos(x) + \cos(x) \cdot \frac{d}{dx} (\cos(x)) \]
   \[ = (\sin(x)) \cos(x) + \cos(x)(-\sin(x)) = -2 \cos(x) \sin(x) \]

3. \[ \frac{d}{dx} \left( \frac{x + 1}{x + 2} \right) = \frac{(x + 2) - (x + 1)}{(x + 2)^2} = \frac{1}{(x + 2)^2} \]

4. Putting \( u = \sin x, \ y = \ln u, \)
   \[ \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{u} \cdot (\cos x) = \frac{\cos x}{\sin x} = \cot x \]

5. Putting \( u = e^x + 1, \ y = \sqrt{u}, \)
   \[ \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{2\sqrt{u}} \cdot e^x = \frac{e^x}{2\sqrt{e^x + 1}} \]