Feedback on the 2013 MATH43001/63001 Exam

A1: (i) Some students claimed that $f(w_1)$ wasn't a term because (as can be proved by induction on the length of terms) if a bound variable occurs in a term then it must be preceded by a quantifier, $\exists$ or $\forall$. This is right, but pretty stupid since quantifiers never occur in terms either.

(ii) & (iii). Done well generally.

(iv) Some students said correctly that this was not a formula, but gave the incorrect reason that it was because w_1 did not occur again after the $\exists w_1$. As I pointed out at the revision lectures there's nothing in L1-4 to say that it has to. I can only assume these students didn't attend the revision lectures.

(v), (vi) & (vii). Done well.

(viii) & (ix). Well done in general although as usual some of the suggested formulae θ didn't even mention any of the free variable x_1, x_2, or instead used n, m which of course are not part of the language.

(x), (xi). Few students got the marks here, and some bizarre 'formulae' were suggested, expressions involving for example $f(\exists w_1\, R(x_1, w_i), f(x_1))$!!!

(xii) Done well.

A2. Generally writing down the particular formal proof was well done *but* a surprisingly large proportion of the students were unable to give a definition of a 'formal proof' !! This was particularly strange since the majority of the MATH33001 students were clearly prepared for this question.

A4. (a) Many students tried to prove this by contradiction assuming $M \models \forall w_1 \exists w_2\, (R(w_1, w_2) \vee R(w_2, w_1))$ but not $M \models \exists w_1\, R(w_1, w_1)$. This was OK but it was simpler just to argue directly from $M \models \forall w_1 \exists w_2\, (R(w_1, w_2) \vee R(w_2, w_1))$ to $M \models \exists w_1\, R(w_1, w_1)$. A surprisingly common mistake in answering this part was to specify a particular structure M and demonstrate that both $M \models \forall w_1 \exists w_2\, (R(w_1, w_2) \vee R(w_2, w_1))$ and $M \models \exists w_1\, R(w_1, w_1)$. This isn't worth anything.

Finally, only a minority of the students had the last part, 'is $R(x_1, x_1) \equiv R(x_2, x_2)$?', correct, yet it follows straight from the definition of $\equiv$ that they are not.

A5. A common mistake here was to propose a 'structure' M for L for which f^M gave some values outside $|M|$, or P^M was not a subset of $|M|$. Some students also spent an inordinate time in explaining why their structures provided the required counter-example, it's really enough just to give the structure and say which of (i),(ii),(iii) are true in it.

B6. Generally well done except everyone missed the tricky bit when you go from $M \models \exists w_1\, \phi(w_1, \pi(b_1), \ldots, \pi(b_n))$ to $M \models \phi(\pi(b), \pi(b_1), \ldots, \pi(b_n))$ for some $b \in |M|$. This isn't immediate and an argument was required here.

B7. Embarrassingly well done, I'll have to set harder formal proofs next year!

B8. Most students knew how to approach this problem but only one got the required Γ correct.