**Feedback on the 2013 MATH43001/63001 Exam**

**A1**: (i) Some students claimed that $f(w_1)$ wasn't a term because (as can be proved by induction on the length of terms) if a bound variable occurs in a term then it must be preceded by a quantifier, $\exists$ or $\forall$. This is right, but pretty stupid since quantifiers never occur in terms either.

(ii) & (iii). Done well generally.

(iv) Some students said correctly that this was not a formula, but gave the incorrect reason that it was because $w_1$ did not occur again after the $\exists w_1$. As I pointed out at the revision lectures there's nothing in L1-4 to say that it has to. I can only assume these students didn't attend the revision lectures.

(v), (vi) & (vii). Done well.

(viii) & (ix). Well done in general although as usual some of the suggested formulae $\theta$ didn't even mention any of the free variable $x_1, x_2$, or instead used $n, m$ which of course are not part of the language.

(x), (xi). Few students got the marks here, and some bizarre 'formulae' were suggested, expressions involving for example $f(\exists w_1\, R(x_1, w_i), f(x_1))$ !!!

(xii) Done well.

**A2.** Generally writing down the particular formal proof was well done *but* a surprisingly large proportion of the students were unable to give a definition of a 'formal proof' !! This was particularly strange since the majority of the MATH33001 students were clearly prepared for this question.

**A4.** (a) Many students tried to prove this by contradiction assuming $M \models \forall w_1 \exists w_2\, (R(w_1, w_2) \vee R(w_2, w_1))$ but not $M \models \exists w_1\, R(w_1, w_1)$. This was OK but it was simpler just to argue directly from $M \models \forall w_1 \exists w_2\, (R(w_1, w_2) \vee R(w_2, w_1))$ to $M \models \exists w_1\, R(w_1, w_1)$. A surprisingly common mistake in answering this part was to specify a particular structure $M$ and demonstrate that both $M \models \forall w_1 \exists w_2\, (R(w_1, w_2) \vee R(w_2, w_1))$ and $M \models \exists w_1\, R(w_1, w_1)$. This isn't worth anything.

Finally, only a minority of the students had the last part, 'is $R(x_1, x_1) \equiv R(x_2, x_2)$ ?', correct, yet it follows straight from the definition of $\equiv$ that they are not.

**A5.** A common mistake here was to propose a 'structure' $M$ for $L$ for which $f^M$ gave some values outside $|M|$, or $P^M$ was not a subset of $|M|$. Some students also spent an inordinate time in explaining why their structures provided the required counter-example, it's really enough just to give the structure and say which of (i),(ii),(iii) are true in it.

**B6.** Generally well done except everyone missed the tricky bit when you go from $M \models \exists w_1\, \phi(w_1, \pi(b_1), \ldots, \pi(b_n))$ to $M \models \phi(\pi(b), \pi(b_1), \ldots, \pi(b_n))$ for some $b \in |M|$. This isn't immediate and an argument was required here.

**B7.** Embarrassingly well done, I'll have to set harder formal proofs next year!

**B8.** Most students knew how to approach this problem but only one got the required $\Gamma$ correct.