Feedback on the 2013 MATH33001 Exam

A1 This was not an easy question to mark because of the variety of ways the statements were understood – compounded by the fact that many students were doubtful that swans really did lay eggs, and whether or not swans did they were quite sure that there were some viviparous birds which did not!

The most common error in answering this question was to formalize 'Some birds lay eggs' as $\exists w_1 (B(w_1) \to E(w_1))$ (where $B(x_1)$ stands for 'x_1 is a bird' etc.). This is actually saying 'There is something which, if it is a bird then it lays eggs', so would be witnessed by a teapot, for example. The correct formalization is $\exists w_1 (B(w_1) \wedge E(w_1))$. On the other hand 'swans are birds' was frequently formalized as $S(x_1) \to B(x_1)$, which is a formula so doesn't have a truth value until the free variable x_1 is interpreted.

A2: (i) Many students claimed that $f(w_1)$ wasn't a term because (as can be proved by induction on the length of terms) if a bound variable occurs in a term then it must be preceded by a quantifier, $\exists$ or $\forall$. This is right, but pretty stupid since quantifiers never occur in terms either.[1]

(ii) & (iii). Done well generally.

(iv) A lot of students said correctly that this was not a formula, but gave the incorrect reason that it was because w_1 did not occur again after the $\exists w_1$. As I pointed out at the revision lectures there's nothing in L1-4 to say that it has to. I can only assume these students didn't attend the revision lectures.

(v) & (vi). Done well.

(vii). Lots of students got this down to an equivalent statement of the form

For some $n \in \mathbb{N}^+$ it is the case that for any $m, k \in \mathbb{N}^+$, if $m|n$ and $k|n$ then either $m|k$ or $k|m$.

They then *chose* n, m, k such that $m|n$, $k|n$ but $m \nmid k$ and $k \nmid m$, for example $n = 10, m = 2, k = 5$ But the n here isn't yours to choose arbitrarily. It is enough here that it holds for some n, it doesn't have to hold for *any* n you happen to think of. And it does hold for some n, eg. $n = 2$, and then any m, k.

(viii) & (ix). Well done in general although as usual some of the suggested formulae θ didn't even mention any of the free variable x_1, x_2, or instead used n, m which of course are not part of the language.

(x), (xi). Few students got the marks here, and some bizarre 'formulae' were suggested, expressions involving for example $f(\exists w_1 R(x_1, w_i), f(x_1))$!!!

(xii) Done well.

A3. Generally well done. Most common error in the definition of a proof was to omit that the Γ_i must be finite. Although I generously didn't subtract any marks many students just said

[1] It reminded me of the nursery rhyme:

A man in the wilderness asked of me
How many strawberries grow in the sea?
I answered him as best I could,
As many red herrings as swim in the wood.

'for $j_1, \ldots, j_s < i$' instead of 'for *some* $j_1, \ldots, j_s < i$'. (Normally if you omit the 'some' then it's read as 'all'.)

In the formal proof a common waste of time, though it wasn't an error, was to have lines

$$\begin{array}{llll}
1. & \forall w_1\, P(w_1) & | & \forall w_1\, P(w_1) \quad \text{REF} \\
2. & \forall w_1\, P(w_1) & | & P(x_1) \quad \forall\text{O}, 1 \\
3. & \forall w_1\, P(w_1) & | & \forall w_2\, P(w_2) \quad \forall\text{I}, 2 \\
4. & \forall w_1\, P(w_1) & | & P(x_2) \quad \forall\text{O}, 3
\end{array}$$

when line (4) could have been obtained directly from line 1 by $\forall$O.

An error which was punished however was to make the step

$$\begin{array}{llll}
n. & \forall w_1\, P(w_1) & | & \forall w_1\, P(w_1) \\
m. & \forall w_1\, P(w_1) & | & \forall w_2\, P(w_2) \\
 & \forall w_1\, P(w_1) & | & \forall w_1 \forall w_2\, (P(w_1) \wedge P(w_2)) \quad \text{AND}, n, m,
\end{array}$$

this isn't a correct use of AND.

A4. (a) Many students tried to prove this by contradiction assuming $M \models \forall w_1 \exists w_2\, (R(w_1, w_2) \vee R(w_2, w_1))$ but not $M \models \exists w_1\, R(w_1, w_1)$. This was OK but it was simpler just to argue directly from $M \models \forall w_1 \exists w_2\, (R(w_1, w_2) \vee R(w_2, w_1))$ to $M \models \exists w_1\, R(w_1, w_1)$. A surprisingly common mistake in answering this part was to specify a particular structure M and demonstrate that both $M \models \forall w_1 \exists w_2\, (R(w_1, w_2) \vee R(w_2, w_1))$ and $M \models \exists w_1\, R(w_1, w_1)$. This isn't worth anything.

A second mistake was to start by assuming $M \not\models \forall w_1 \exists w_2\, (R(w_1, w_2) \vee R(w_2, w_1))$ and $M \models \exists w_1\, R(w_1, w_1)$ and try to show a contradiction. Maybe some one should try to explain to me whyever anyone would do this!

Finally, almost no one had the last part, 'is $R(x_1, x_1) \equiv R(x_2, x_2)$?', correct, yet it follows straight from the definition of $\equiv$ that they are not.

B5. A common mistake here was to propose a 'structure' M for L for which f^M gave some values outside $|M|$. There was also a tendency to propose a structure, find half way through trying to show that it did the job, that it didn't, and then go back and change some part of it ignoring the now invalid intermediate conclusions! Final some students tried this question without actually having the faintest idea what to do!

B6. Disappointing, the highest mark was 9 out of 12 and most students scored about 3. There was a widespread tendency to apply rules incorrectly and to invent new 'imaginary rules' because they suited the job at hand, despite the correct rules being given in the appendix. Often these purported 'proofs' did not actually use the left hand side $\forall w_1 \forall w_2\, (P(w_1) \vee Q(w_2))$, as if I'd stuck it there as a sort of red herring!!

B7. No one got anywhere near answering this question correctly. Those who attempted it seemed to assume that it was last year's question in disguise (even to the extent of reproducing, as best they could, last year's solution) and identified 'finite separation' with 'finite'. This misunderstanding even extended to the statement of the Compactness Theorem – the question asked for the Compactness Theorem for a relational language but many student stated the Compactness Theorem for normal structures (as was requested last year).