Feedback on the 2012 MATH33001 exam

A1. Most students correctly said that the conclusion followed from the premises but failed to point out that the reason for this was that the argument did not depend on the meaning of 'frog', 'sings' etc..

A2. (i) Nearly everyone correctly said it was not a term of L but justified this by saying that 'whenever a bound variable appears in a term then so must one of $\exists$, $\forall$. Well, that's vacuously true and it got the marks but it was rather a silly statement since quantifiers *never* appear in terms, nor do bound variables!!

(iv) Most students realized there was something wrong with the brackets but few students could actually specify a property that all formulae have but this expression did not possess (see model answers).

(v)-(vii) A quite common error was to give formulae for $\theta_1(x_1, x_2)$, *etc.* which didn't actually mention x_1, x_2 or indeed any free variables at all. Only the perpetrators can know why the did this, I certainly can't explain it! A second, inexplicable, though quite common, error was to suddenly make R a *binary* relation symbol at this point, with $R^M = \{\langle n, m\rangle \mid n < m\}$. Generally students seemed to find these parts of the question hard, very few had correct answers for all four formulae.

A3. Despite my repeatedly saying that you should learn up the key definitions some students still could not reproduce a correct definition of a proof. There was also some confusion about the definition of a 'formal proof' and of 'proves', as in $\Gamma \vdash \theta$. The proof I asked for was almost uniformly well done, even by students who couldn't say what a proof was!

A4. Part (a) was generally well done but lots of students when awry on part (b), arguing as follows:

Let M be an arbitrary structure for L. Then

$$\begin{aligned}
M \models \forall w_1\,(P(w_1) \vee Q(w_1)) &\iff \forall a \in |M|,\ M \models P(a) \text{ or } M \models Q(a)\\
&\iff \forall a \in |M|,\ M \models P(a) \text{ or for}\\
&\qquad \text{some } b \in |M|,\ M \models Q(b)\\
&\iff M \models \forall w_1\, P(w_1) \text{ or } M \models \exists w_1\, Q(w_1)\\
&\iff M \models \forall w_1\, P(w_1) \vee \exists w_1\, Q(w_1).
\end{aligned}$$

What I wanted to see was some explanation of where, from left to right, the second line came from. It looked to me as if the student had just written down what they knew they had to get to. I might have concluded that the student was so smart that they thought this was just too obvious (and so didn't need any explanation) – except that the $\iff$ on this line is actually incorrect, which dashed any thoughts I might have entertained about the elevated level of their smartness!!

B5. Almost everyone did this question and generally scored quite well, even though in these part B questions I wasn't tolerant of mistakes as I tended to be in part A. Mostly the students who went wrong did so because they chose structures which were too complicated, for example $|M| = \mathbb{N}$, $R^M = \{\langle n, m\rangle \in \mathbb{N}^2 \mid m \text{ divides } n\}$, for which it wasn't always so easy to see whether (i),(ii),(iii) held or not. [For example does 0 divide 0? – according to the definition of 'divides' it does but I think several students thought it did not.] It was a much safer bet to construct structures where $|M|$ was some small finite set, such as $\{0, 1\}$.

B6. Despite almost everyone trying it few students got this right, it was very much an all or nothing question. One *very* common error (despite the warning given by the Coursework) was to incorrectly apply $\exists O$ to $\Gamma, \neg P(x_1) \wedge Q(x_1) \mid \neg P(x_1)$ (or similar) to get to $\Gamma, \exists w_1\, (\neg P(w_1) \wedge Q(w_1)) \mid \neg P(x_1)$.

B7. Few students answered this question. Most of those who made a serious attempt got full marks.