Two hours

THE UNIVERSITY OF MANCHESTER

MATHEMATICS 0C1/1C1

21st January 2013
14.00 – 16.00

Answer SIX of the EIGHT questions
If more than SIX questions are attempted then credit will be given for the FIRST SIX answers.

The use of calculators is not permitted
1. (1) Multiply out the brackets from the following expressions and collect terms.
   (i) \((x^2 - 3)(x + 4)\)
   (ii) \((a - b + 1)(a + b - 1)\)
   (iii) \((2 - x)(1 - (x - 2))\)
   (iv) \((1 - 2x)(x - 1)^2\)  

   (2) In 1(iv) above what is the term in \(x^2\)? What is the coefficient of \(x\)?
   What is the constant term?

   (3) Express each of the following in the form \(x^k\) where \(k\) is an integer or a rational number in its simplest form:
   (i) \(\frac{x^4}{x^8}\)
   (ii) \(x^{-2}\sqrt{x}\)
   (iii) \((x^4)^{5/6}\)

2. Solve the following equations for \(x\). (Find all solutions.)
   (1) \(x^2 - 6x + 8 = 0\)
   (2) \(5x^2 + 4x - 2 = 3x^2 + x - 1\)
   (3) \(\frac{x + 5}{x - 5} = \frac{x - 2}{2}\)
   (4) \(\frac{2}{x + 2} - \frac{1}{x} = \frac{1}{x - 4}\)
   (5) \((x + 1)^4 - 5(x + 1)^2 + 4 = 0\)

3. Solve the following equations for \(x\). (Find all solutions.)
   (i) \(9^x = 3\)
   (ii) \(\log_3 \left( \frac{2}{x + 8} \right) = -2\)
   (iii) \(\log_3 (9^{x+1}) = x\)
   (iv) \(x \log_x (3) = \log_x (2)\)
   (v) \(\log_x (4x - 4) = 2\)
4. Find the equation of the line \( C \) passing through the points \((-2, 1)\) and \((1, 7)\). [2 marks]

(2) Does the point \((3, 10)\) lie on this line? [1 mark]

(3) At what point \( A \) does the line \( C \) intersect the line \( y = 1 - 2x \)? [2 marks]

(4) Is the line \( y = 1 - 2x \) perpendicular to the line \( C \)? [1 mark]

(5) What is the distance between the points \( A \) and \((0, 5)\)? [2 marks]

(6) What is the cosine of the angle between the line \( C \) and the \( x \)-axis? [2 marks]

5. Consider the curves \( C \) and \( D \) given by \( y = x^2 - 2 \) and \( y = 2x^2 + 7x - 2 \) respectively.

(1) Find the two points where these curves cross. [3 marks]

(2) At what value of \( x \) do these two curves have the same slope? [2 marks]

(3) Show that the point \((-1, -7)\) is on the curve \( D \) and find the equation of the tangent to \( D \) at that point. [3 marks]

(4) At what points does this tangent to \( D \) at \((-1, -7)\) cross the curve \( C \)? [2 marks]

6. (a) Let \( f(x) = \frac{2}{x} + 1 \).

(1) What is the domain of \( f \)? [1 mark]

(2) What is \( f(f(x)) \)? [2 marks]

(3) Find \( f^{-1}(x) \). [2 marks]

(b) The right angled triangle below has hypothenuse of length 6 and \( \cos(A) = 2/3 \).

\[ \begin{array}{c}
6 \\
A \\
b
\end{array} \]

Find:

(1) \( b \) (2) \( \sin(A) \) (3) \( \cot(A) \) (4) \( \cos(A/2) \) (5) \( \cos(A - \pi/4) \) [1 mark for each part]

3 of 4
7. (1) Differentiate the following functions
   
   (i) \( y = 6x^6 - 6 \)
   
   (ii) \( y = x^{-4/3} \)
   
   (iii) \( y = e^{1-2x} \)  
   
   [1 mark each]

   (2) Find and classify the two stationary points of the function \( f(x) = x^3 - 6x^2 + 9x \).  
       [4 marks]
       
       Sketch the graph of this function and using this graph indicate why the equation
       
       \[ x^3 - 6x^2 + 9x = 6 \]
       
       has only one solution.  
       
       [3 marks]

8. Differentiate the following functions

   (1) \( y = (x^2 + 1)^{-3} \)
   
   (2) \( y = \sin(x) \cos(2x) \)
   
   (3) \( y = \frac{1-x}{1+x} \)
   
   (4) \( y = \ln(2 + \sin(x)) \)
   
   (5) \( y = 2e^{\sqrt{x}} \)
   
   [2 marks each]