

Two hours

**THE UNIVERSITY OF MANCHESTER**

MATHEMATICS 0C1/1C1

21st January 2013

14.00 – 16.00

Answer SIX of the EIGHT questions

If more than SIX questions are attempted then credit  
will be given for the FIRST SIX answers.

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**The use of calculators is not permitted**

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1. (1) Multiply out the brackets from the following expressions and collect terms.

(i)  $(x^2 - 3)(x + 4)$

(ii)  $(a - b + 1)(a + b - 1)$

(iii)  $(2 - x)(1 - (x - 2))$

(iv)  $(1 - 2x)(x - 1)^2$

[4 marks]

(2) In 1(iv) above what is the term in  $x^2$ ? What is the coefficient of  $x$ ?  
What is the constant term?

[3 marks]

(3) Express each of the following in the form  $x^k$  where  $k$  is an integer or a rational number in its simplest form:

(i)  $\frac{x^4}{x^8}$       (ii)  $x^{-2}\sqrt[3]{x}$       (iii)  $(x^4)^{5/6}$

[3 marks]

2. Solve the following equations for  $x$ . (Find *all* solutions.)

(1)  $x^2 - 6x + 8 = 0$

(2)  $5x^2 + 4x - 2 = 3x^2 + x - 1$

(3)  $\frac{x + 5}{x - 5} = \frac{x - 2}{2}$

(4)  $\frac{2}{x + 2} - \frac{1}{x} = \frac{1}{x - 4}$

(5)  $(x + 1)^4 - 5(x + 1)^2 + 4 = 0$

[2 marks for each part]

3. Solve the following equations for  $x$ . (Find *all* solutions.)

(i)  $9^x = 3$

(ii)  $\log_3 \left( \frac{2}{x + 8} \right) = -2$

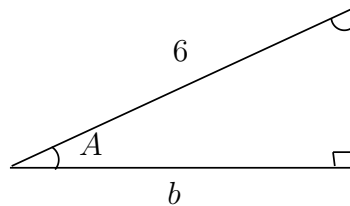
(iii)  $\log_3 (9^{x+1}) = x$

(iv)  $x \log_x (3) = \log_x (2)$

(v)  $\log_x (4x - 4) = 2$

[2 marks for each part]

4. Find the equation of the line  $\mathcal{C}$  passing through the points  $(-2, 1)$  and  $(1, 7)$ . [2 marks]
- (2) Does the point  $(3, 10)$  lie on this line? [1 mark]
- (3) At what point  $A$  does the line  $\mathcal{C}$  intersect the line  $y = 1 - 2x$ ? [2 marks]
- (4) Is the line  $y = 1 - 2x$  perpendicular to the line  $\mathcal{C}$ ? [1 mark]
- (5) What is the distance between the points  $A$  and  $(0, 5)$ ? [2 marks]
- (6) What is the cosine of the angle between the line  $\mathcal{C}$  and the  $x$ -axis? [2 marks]
5. Consider the curves  $\mathcal{C}$  and  $\mathcal{D}$  given by  $y = x^2 - 2$  and  $y = 2x^2 + 7x - 2$  respectively.
- (1) Find the two points where these curves cross. [3 marks]
- (2) At what value of  $x$  do these two curves have the same slope? [2 marks]
- (3) Show that the point  $(-1, -7)$  is on the curve  $\mathcal{D}$  and find the equation of the tangent to  $\mathcal{D}$  at that point. [3 marks]
- (4) At what points does this tangent to  $\mathcal{D}$  at  $(-1, -7)$  cross the curve  $\mathcal{C}$ ? [2 marks]
6. (a) Let  $f(x) = \frac{2}{x} + 1$ .
- (1) What is the domain of  $f$ ? [1 mark]
- (2) What is  $f(f(x))$ ? [2 marks]
- (3) Find  $f^{-1}(x)$ . [2 marks]
- (b) The right angled triangle below has hypotenuse of length 6 and  $\cos(A) = 2/3$ .



Find:

- (1)  $b$       (2)  $\sin(A)$       (3)  $\cot(A)$       (4)  $\cos(A/2)$       (5)  $\cos(A - \pi/4)$  [1 mark for each part]

7. (1) Differentiate the following functions

(i)  $y = 6x^6 - 6$

(ii)  $y = x^{-4/3}$

(iii)  $y = e^{1-2x}$

[1 mark each]

(2) Find and classify the two stationary points of the function  $f(x) = x^3 - 6x^2 + 9x$ . [4 marks]

Sketch the graph of this function and using this graph indicate why the equation

$$x^3 - 6x^2 + 9x = 6$$

has only one solution.

[3 marks]

8. Differentiate the following functions

(1)  $y = (x^2 + 1)^{-3}$

(2)  $y = \sin(x) \cos(2x)$

(3)  $y = \frac{1-x}{1+x}$

(4)  $y = \ln(2 + \sin(x))$

(5)  $y = 2e^{\sqrt{x}}$

[2 marks each]

**END OF EXAMINATION PAPER**