Two hours

THE UNIVERSITY OF MANCHESTER

MATHEMATICS 0C1/1C1

16th January 2012

9.45 – 11.45

Answer **SIX** questions. If you answer more than six questions only the first six appearing in your answer book will be marked.

The use of calculators is not permitted

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1. (1) Multiply out the brackets from the following expressions and collect terms.

(i) $(x^2 - 3)(x + 5)$

(ii) $(a - b + 2)(a + b - 2)$

(iii) $(2 - x)(2 - (x - 1))$

(iv) $x(1 - 2x)(x - 1)$

[4 marks]

(2) In 1(iv) above what is the term in x^2 ? What is the coefficient of x ? What is the constant term?

[3 marks]

(3) Express each of the following in the form x^k where k is a rational number in its simplest form:

(i) $\dfrac{x^3}{x^6}$ (ii) $x^{-1}\sqrt[4]{x}$ (iii) $(x^6)^{1/4}$

[3 marks]

2. Solve the following equations for x. (Find *all* the solutions.)

(1) $x^2 - 3x - 10 = 0$

(2) $3x^2 + 4x - 2 = x^2 + x - 1$

(3) $\dfrac{x+2}{x-4} = \dfrac{x-1}{2}$

(4) $\dfrac{2}{x+6} - \dfrac{1}{x+4} = \dfrac{1}{x}$

(5) $(x^2 + 1)^2 - 5(x^2 + 1) + 6 = 0$

[2 marks for each part]

3. (1) Solve the following equations for x. (Find *all* solutions.)

(i) $16^x = 4$

(ii) $\log_3\left(\dfrac{2}{x-3}\right) = -1$

(iii) $\log_2\left(4^{x-1}\right) = x + 1$

(iv) $x\log_x(2) = \log_x(3)$

(v) $\log_x(x^3 + x - 11) = 3$

[2 marks for each part]

4. (1) Find the equation of the line $\mathcal{C}$ passing through the points $(-1, 2)$ and $(1, 6)$. [2 marks]

(2) Show that the point $(2, 8)$ lies on this line. [1 mark]

(3) At what point A does the line $\mathcal{C}$ cross the x axis? [1 marks]

(4) What is the distance between the points A and $(1, 6)$? [2 mark]

(5) By considering the triangle formed from the points A, $(1, 6)$ and $(1, 0)$ find the sine of the angle between the line $\mathcal{C}$ and the x axis. [2 marks]

(6) Find the point of intersection of the line $\mathcal{C}$ with the line $y = 16 - 2x$. [2 marks]

5. Let $\mathcal{C}$ be the curve $y = x^2 - x - 3$ and let $\mathcal{E}$ be the line $y = x - 4$.

(1) Find the point A where $\mathcal{E}$ intersects $\mathcal{C}$. [2 marks]

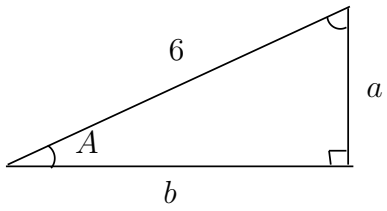
(2) Show that $\mathcal{E}$ is the tangent to $\mathcal{C}$ at A. [2 marks]

(3) Find the equation of the normal to $\mathcal{E}$ at A. [2 marks]

(4) Find the other point B at which this normal intersects $\mathcal{C}$ [2 marks]

(5) Find the equation of the tangent to $\mathcal{C}$ at B. [2 marks]

6. The right angled triangle below has hypothenuse of length 6 and $\cos(A) = 3/4$.



Find:

(1) b (2) $\sin(A)$ (3) a (4) $\tan(A)$ (5) $\cos(-A)$

(6) $\cos(2A)$ (7) $\sin(2A)$ (8) $\cos(A/2)$ (9) $\cos(3A)$ (10) $\sin(A + \pi/4)$

[1 mark for each part]

7. (1) Differentiate the following functions

(i) $y = 2x^9 - 9$

(ii) $y = \sqrt[4]{x}$

(iii) $y = e^{2x+1}$ [1 mark each]

(2) Find and classify the two stationary points of the function

$$f(x) = 2x^3 + 3x^2 - 12x.$$ [4 marks]

Sketch the graph of this function and using this graph indicate why the equation

$$2x^3 + 3x^2 - 12x + 8 = 0$$

has only one solution. [3 marks]

8. Differentiate the following functions

(1) $y = (2x + 1)^{-2}$

(2) $y = \sin^2(x)$

(3) $y = \dfrac{1 + x}{1 - x}$

(4) $y = \ln(1 + e^x)$

(5) $y = \sqrt{\cos(x) + 1}$

[2 marks each]