Two hours

THE UNIVERSITY OF MANCHESTER

MATHEMATICS 0C1/1C1

18th January 2010
9.45 – 11.45

Answer SIX questions

The use of calculators is not permitted
1. (1) Remove the brackets from the following expressions by multiplying out.
   (i) \((x^2 - 1)(x + 5)\)
   (ii) \((a - b - 1)(a + b - 2)\)
   (iii) \((1 - a)(b - (a - 1))\)
   (iv) \(x(x + 1)(1 - 3x)\)

(2) In 1(iv) above what is the term in \(x^2\)? What is the coefficient of \(x\)? What is the constant term?

(3) Express each of the following in the form \(x^k\) where \(k\) is a rational number:
   (i) \(\frac{x^3}{x^6}\)
   (ii) \(x^{\sqrt{3}}\)
   (iii) \((x^6)^{1/3}\)

2. Solve the following equations for \(x\). (Find all the solutions.)
   (1) \(x^2 - 6x + 8 = 0\)
   (2) \(4x^2 + 2x - 3 = 2x^2 - x - 2\)
   (3) \(\frac{x + 2}{x - 2} = \frac{x}{3}\)
   (4) \(\frac{1}{3 - x} + \frac{1}{x + 1} = \frac{-2}{x + 9}\)
   (5) \(2^x - 2^{x+1} + 1 = 0\)

3. (1) Solve the following equations for \(x\). (Find all solutions.)
   (i) \(8^x = 2\)
   (ii) \(\log_3 \left( \frac{27}{x + 1} \right) = 2\)
   (iii) \(\log_2 (4^{x+1}) = x\)
   (iv) \(\log_x (x^3 - 2x + 1) = 3\)

(2) The volume \(V(t)\) of porridge in a cooking pot at time \(t\) is related to the volume \(V_0\) at time 0 by the equation \(V(t) = V_0a^t\) for some \(a > 1\). If the volume doubles every 3 minutes what must \(a\) be? If the volume of the cooking pot is 5\(V_0\) how long is it before the pot overflows?
4. (1) Find the equation of the line $C$ passing through the points $(-3, -3)$ and $(1, 9)$. [2 marks]

(2) Show that the point $(-4, -6)$ lies on this line. [1 mark]

(3) At what point $A$ does the line $C$ cross the $x$ axis? At what point $B$ does the line $C$ cross the $y$ axis? [2 marks]

(4) What is the distance between the points $A$ and $B$? [1 mark]

(5) By considering the triangle formed from the points $A$, $B$ and $(0, 0)$ find the sine of the angle between the line $C$ and the $x$ axis. [2 marks]

(6) Find the point of intersection of the line $C$ with the line $y = 11x - 10$. [2 marks]

5. Consider the curves $D$ and $E$ given by $y = 2x^2 + x - 1$ and $y = x^2 + 2x + 1$ respectively.

(1) Find the two points where these curves cross. [3 marks]

(2) At what value of $x$ do these two curves have the same slope? [2 marks]

(3) Show that the point $(-2, 1)$ is on the curve $E$ and find the equation of the tangent to $E$ at that point [3 marks]

(4) Find the equation of the line through $(-2, 7)$ which is normal to this tangent to $E$ at $(-2, 1)$. [2 marks]

6. (1) Differentiate the following functions

(i) $y = 4x^3 + 3$ 

(ii) $y = \sqrt{x}$

(iii) $y = \cos(2x - 1)$ [1 mark each]

(2) Find and classify the two stationary points of the function

$$f(x) = x^3 + 3x^2 - 9x + 5.$$ [4 marks]

Sketch the graph of this function and using this graph indicate why the equation

$$x^3 + 3x^2 - 9x + 6 = 0$$ has only one solution. [3 marks]
7. Differentiate the following functions

(1) \( y = (x - 1)^2e^x \)

(2) \( y = x(1 - \ln x) \)

(3) \( y = \frac{x - 1}{x + 1} \)

(4) \( y = \ln(\cos(x)) \)

(5) \( y = e^{\sqrt{x}} \)

[2 marks each]

8. (1) Find the indefinite integral

\[ \int \left(x + \frac{1}{\sqrt{x}}\right)^2 \, dx. \]

[3 marks]

(2) Evaluate the definite integral

\[ \int_0^{\pi/2} e^x + \cos(x) \, dx \]

[4 marks]

(3) Find the area bounded by the curve \( y = x^2 + 1 \), the \( y \) axis and the lines \( x = -1 \) and \( x = 1 \).

Using this answer, or otherwise, find the area bounded by the curve \( y = x^2 + 1 \) and the line \( y = 2 \).

[3 marks]