Introduction

“Except during the nine months before he draws his first breath, no man manages his affairs as well as a tree does.”

– George Bernard Shaw

The human placenta incorporates almost all functions of the adult body. It substitutes the fetal lung and digestive system and serves as an interface for metabolic exchange between mother and fetus.

What do we know about the placenta?

• It is a fast-growing organ in a changing environment;
• There is great interspecies variability;
• Well-orchestrated and synchronised both spatially and temporally from implantation to labour.

What we do not yet know:

• The maternal vascular distribution in the human placenta;
• The precise mechanisms that govern spatial organisation of the materno-fetal circulation;
• How the placental structure and geometry affect intrauterine haemodynamics, and vice versa.

Applications are:

• Pre-eclampsia, diabetes and other pregnancy complications;
• Intrauterine growth restriction (IUGR) and placental insufficiency.

Objective: development and analysis of a mathematical model for placental circulatory unit, the placentone.

Aims:

• What is the effect of maternal vessels’ position and blood flow rates on the concentration pattern in the placentone?
• Is there an optimal intervillous space density (villous volume fraction) in terms of net metabolic exchange rate?

The Model

We assume the intervillous space to be a homogeneous porous medium perfused by an incompressible Newtonian liquid, representing maternal blood. The placentone is enclosed in an impermeable hemisphere at the circular base of which (S1) are a central source of flux (spiral artery) and two sinks (decidual veins), aligned along a diameter.

Steady axisymmetric fluid motion about the z-axis and advection-dominated solute transport are modelled by Darcy’s law and a convection-reaction equation, subject to corresponding boundary conditions (in dimensionless form):

\[ \nabla \cdot \mathbf{u} = 0, \quad \mathbf{u} = -\nabla P, \quad \text{in } V \]

\[ (\mathbf{u} \cdot \nabla)C = -D_a f(C), \quad \text{in } V \]

\[ \lim_{r \to 0} r_n u = \frac{1}{2} (\delta(z) - \frac{1}{2} [\delta(z-a) + \delta(z+a)]), \]

\[ u \cdot n = 0 \text{ on } S_2, \]

\[ C|_{r=r_{cut}} = 1. \]

/ defines uptake kinetics: \( C = C^* (1^{\text{st}} \text{ order}), \quad f(C) = C^* - C \) (2^{nd} order), \( f(C) = C^* - e^{C-C^*} \) (Hill-type).

Two non-dimensional parameters

\[ \text{Relative artery-vein distance: } h_0 = \frac{a}{L} \]

\[ \text{Darcikeller number: } Da = \frac{a}{q_0} \]

\[ \text{Peclet number: } Pe = \frac{a^2}{D C^*} \sim 1 \]

The Method of Images

We use the method of images, based on Butler and Weiss’ Sphere theorems [9], to find exact flow and pressure fields in a closed form. The image system for three point singularities inside the hemisphere consists of two point and two line sinks.

Results

We find how a solute is absorbed along the streamlines given by analytical flow field. We also introduce an integral measure of the uptake efficiency – the net uptake rate and study a role of the villous volume fraction.

Conclusions

We have derived and investigated a simple model for steady flow and nutrient transport in the human placenta.

• The relationship between the solute consumption rate, the arterial flow rate and the size of the placenta ensures a homogeneous concentration pattern;
• Effective metabolic exchange is achieved if the decidual veins are located on the periphery of the placenta;
• Our model suggests the existence of an optimal volume fraction of villous tissue irrespective of the type of passive uptake kinetics.

References


Fig. 1: Schematic human placenta at term (reproduced from [1]).

Fig. 2: A diagram of a typical placentone (left), where SA – spiral artery, DV – decidual vein, CC – central cavity, AV – anchoring villi. The spiral artery entering the dense villous tree (right) [2].


Fig. 3: Geometry of the model in cylindrical coordinates.

Physiological data [1,6,8]

L = 1 – 2 cm
q_0 = 5 ml/min
Reynolds number = \( \frac{24 \pi a}{5} \sim 1 \)

Fig. 4: Left: streamlines (blue) and pressure isolines (green) in the placentone at the varying source-sink distance; Top: an illustration of the images system.

Fig. 5: Solute concentration patterns for different values of the model parameters (q_0, L are fixed).

Fig. 6: Effect of artery-vein distance (left) and flow rates in the spiral artery (right) on the solute net uptake rate.

Fig. 7: Net uptake rate vs. villous volume fraction. Left: qualitative dependence for the first-order kinetics. Right: Computed curves.