

**LMS/EPSRC Short Instructional Course on
*Computational Differential Equations*¹
September 11-16, 2005**

MIMS, School of Mathematics, University of Manchester

Organizer: Prof. Nicholas J. Higham

Course web page: <http://www.ma.man.ac.uk/~higham/cde05/>

Differential equations are ubiquitous in science and engineering, being used for all kinds of modelling and prediction. The solutions to most differential equations have no convenient explicit form and hence the *numerical solution* of differential equations is a subject of fundamental importance. Mathematicians working in a wide variety of areas, including continuum mechanics, fluid mechanics, stochastic differential equations, bifurcation theory, dynamical systems, and mathematical biology, all increasingly need expertise in the computational treatment of differential equations.

This course provides an overview of three important topics in the computational solution of differential equations, covering both theoretical aspects and practical computation. MATLAB is chosen as the programming language in view of its ease of use and its widespread adoption in research and education. The course is aimed at first and second year mathematics Ph.D. students working in any area that requires computational solution of differential equations; it assumes a familiarity with numerical analysis but not a strong background in the subject. Experience in programming is assumed, but it is not necessary for the students to be familiar with MATLAB. Each lecturer will provide the students with MATLAB codes for experimentation and modification in the laboratory sessions.

1 Programme

The course begins on Sunday at Hulme Hall. After registration, it opens with a formal lecture on computational differential equations that will set the scene for the course, given by Professor Peter Jimack, of the University of Leeds.

From Monday all lectures and laboratories are held on the top floor of the Ferranti Building, Sackville Street, on the University of Manchester campus (building 20 on the campus map).

Monday morning will consist of an introduction to MATLAB, given by the course organizer, with a combined lecture/laboratory session.

Courses 1–3 will start on Monday afternoon, with one hour introductory lectures. The remaining 6 half-day sessions contain a further 6 contact hours per course, split flexibly between lectures and laboratory sessions. The laboratory sessions will involve the students working through example sheets. The course finishes at Friday lunchtime.

1.1 Course Overview

Course 0: MATLAB (Prof. Nick Higham, University of Manchester)

This mini-course aims to provide the basic grounding in MATLAB that is needed for Courses 1–3 and also introduces the students to the computing environment that they will be using. It treats essentials of MATLAB, including vectors and matrices, graphics, control structures and logical tests, and M-files.

Course 1: Spectral Methods (Prof. Nick Trefethen, University of Oxford)

Spectral methods are numerical methods for ODEs and PDEs based on global rather than local approximations, e.g., Chebyshev or Legendre polynomial or trigonometric interpolants as opposed to piecewise linears or quadratics. For smooth problems posed in simple geometries, a 20-line MATLAB spectral method code can often get the answer to ten digits of accuracy in a few seconds. The lecturer is well known for using MATLAB codes to implement spectral methods through his 2000 SIAM book *Spectral Methods in MATLAB*.

¹Version dated 12-8-05.

Course 2: Stochastic Differential Equations (Prof. Des Higham, University of Strathclyde)

Stochastic differential equations arise in mathematical models of physical systems which possess inherent noise and uncertainty. Such models have been used with great success in a variety of application areas, including biology, epidemiology, mechanics, economics and finance. This course will give a gentle introduction to SDEs and their numerical simulation, without assuming a background in probability theory. The treatment will be at the level of the widely-used article *An algorithmic introduction to numerical simulation of stochastic differential equations*, by D. J. Higham, SIAM Review, Education Section, 43, 2001, 525–546, with extra material on applications, error control and computation of mean exit times.

Course 3: Finite Element Methods (Prof. David Silvester, University of Manchester)

Finite element approximation methods are widely used in scientific and engineering computations. The finite element method is also attractive from a mathematical perspective because of its rigorous underpinning. The basic aim here is to develop the theory of a priori and a posteriori error estimation in the context of elliptic PDEs. The problems discussed will range in complexity from Poisson's equation modelling diffusion phenomena to the nonlinear Navier-Stokes equations modelling steady incompressible fluid flow. The laboratory sessions will introduce the students to the MATLAB software that the lecturer has developed in conjunction with his forthcoming OUP book *Finite Elements and Fast Solvers*.

Timetable

Time	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday
7.30–8.30		Breakfast	Breakfast	Breakfast	Breakfast	Breakfast
9.00–10.00		Course 0	Course 2	Course 3	Course 1	Course 2 /Lab
10.00–11.00		Course 0	Course 3	Course 1	Course 2	Course 3/Lab
11.00–11.30		Coffee	Coffee	Coffee	Coffee	Coffee
11.30–12.30		Course 0	Course 1	Course 2	Course 3	Course 1/Lab
12.30–14.00		Lunch	Lunch	Lunch	Lunch	Lunch
14.00–15.00		Course 1	Lab		Lab	
15.00–16.00	Registration ¹	Course 2	Lab	Free afternoon	Lab	
16.00–16.30	Welcome ¹	Tea	Tea		Tea	
16.30–17.30	Opening Lecture ¹	Course 3	Lab		Lab	
18.30–19.30	Dinner ^a	Dinner ^b	Dinner ^b	Dinner ^b	Dinner ^b	

¹ At Hulme Hall, *Computational Partial Differential Equations: Some Techniques and Applications* by Professor P. Jimack.

^a For all.

^b For Hulme Hall residents only.

Meals

Breakfast and dinner are served at Hulme Hall for the residents. Lunch is provided on campus for all participants.

Registration and Directions to Hulme Hall

Registration will be on Sunday September 11 2005 from 3–4pm at the reception of

Hulme Hall, Oxford Place, Victoria Park, Manchester, M14 5RR
Tel: 0161 275 0210
Fax: 0161 257 3059
E-mail: hulme.hall@manchester.ac.uk

For those staying at at Hulme Hall you can collect your room keys any time after 10.00am on Sunday Morning. In the unlikely event that the reception is closed there is a Duty Tutor number that can be dialled from outside the building and the tutor will come to reception to give you your room keys.

For links to maps, please see the conference web page at

<http://www.ma.man.ac.uk/~higham/cde05/>

For information on how to get to Manchester see

<http://www.maths.man.ac.uk/mims/directions.html>

Hulme Hall is about a 15 minute walk south along Oxford Road from the Oxford Road part of the University campus. The nearest train station is Manchester Oxford Road, from which Hulme Hall is about a 25 minute walk. Buses 41–46 (and many others) take you down Oxford Road and to within 3 minutes walk from Hulme Hall.

Internet Access

Internet access is available from the bedrooms in Hulme Hall, for which there is a charge of £5 per 24 hours.

Free Afternoon

Things to do/places to visit on Wednesday afternoon

- The City Centre for some light (or even heavy) retail therapy.
- Whitworth Art Gallery.
- University Museum (opposite the Maths Tower).
- Manchester City Centre: take (pretty much) any bus going up Oxford Road.
- Museum of Science and Technology.
- The Lowry Arts Centre (Salford Quays, accessible by Metrolink).
- The Imperial War Museum North (Salford Quays, accessible by Metrolink).
- Manchester United Stadium Tour (Old Trafford, accessible by Metrolink).
- Continue to work in the PC lab(!).

For some idea of what to expect at the above locations, see the photos at

<http://www.ma.man.ac.uk/~higham/photos/manchester/index.htm>

Getting to Know Each Other

This course provides an excellent opportunity to meet PhD students with similar research (and other?) interests. Make an effort to find out what people are working on.

To facilitate scientific (and non-scientific!) discussions, the Hulme Hall bar will be open every night until 10.30 (Sunday) or 11 (Monday–Thursday).

Further Information

If you require any further information before the start of the course, please contact the course organiser, Professor Nick Higham (Phone: +44 (0)161 275 5822, higham@ma.man.ac.uk) or his secretary Miss Louise Stait (Phone: +44 (0)161 306 3641, louise@maths.man.ac.uk).