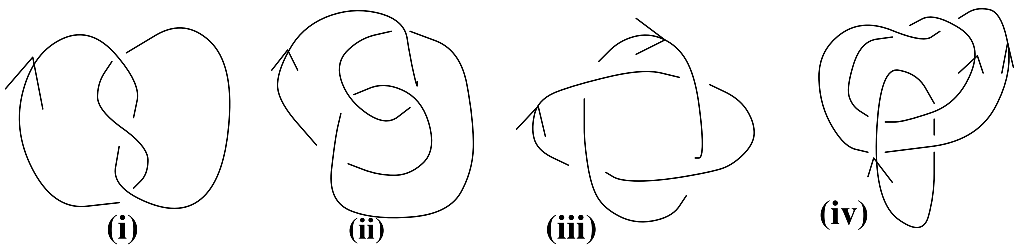
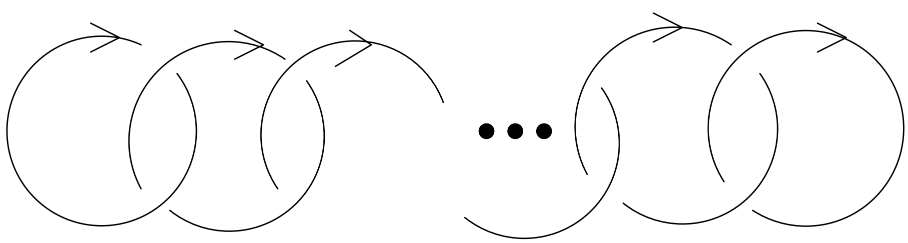
MT3522 Knot Theory Examples 5 2004/2005

1. Show that (i) for the negative Hopf link H^-, $f_{H^-}(A) = -A^{10} - A^2$, and (ii) for the negative trefoil knot T^-, $f_{T^-}(A) = -A^{16} + A^{12} + A^4$. (Thus the Jones polynomial distinguishes between the two Hopf links and also between the two trefoil knots.)

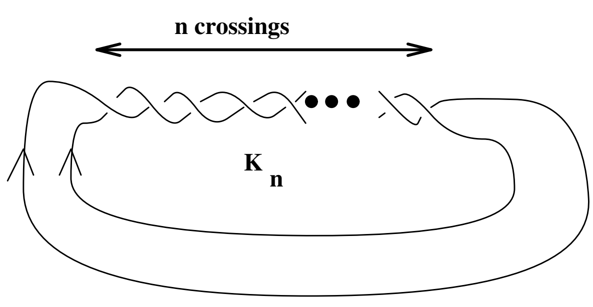
2. Calculate the Jones polynomial $f_K(A)$ for each of the knots K and links L in Examples 3, Question 3. (Diagrams repeated below.)



3. Let $L(n)$ denote the n-link negative chain shown below. Use induction on n to calculate its Jones polynomial $f_{L(n)}$.



4. Let K_n be the link with n crossings of Examples 3, Question 6.



Prove the recursion relation

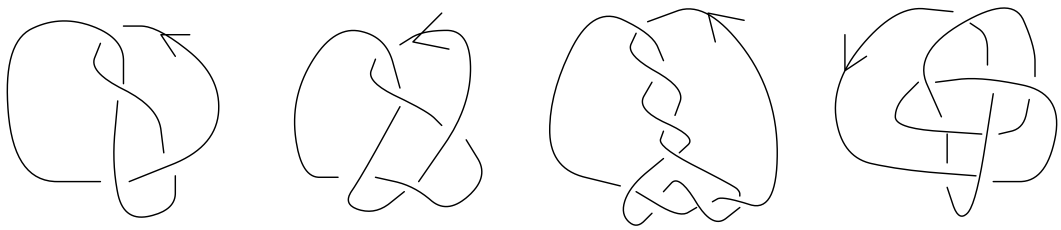
$$A^4 f_{K_n}(A) - A^{-4} f_{K_{n-2}}(A) = (A^{-2} - A^2) f_{K_{n-1}}(A).$$

Let $a_n = f_{K_n}(e^{i\pi/12})$. Show that

$$a_n = \left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)a_{n-1} + \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)a_{n-2},$$

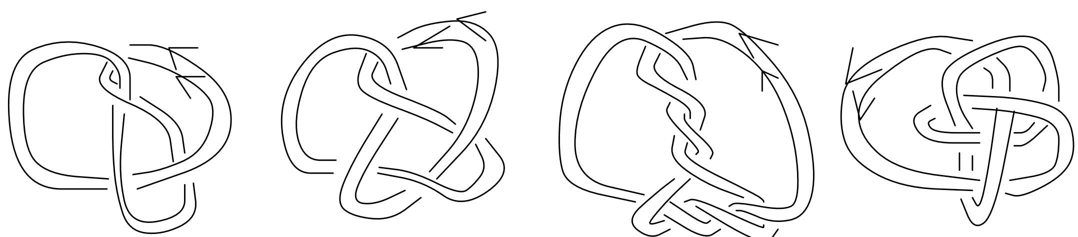
and hence evaluate a_n for all positive integers n.

5. **(i)** Calculate the writhe of each of the following knot diagrams.



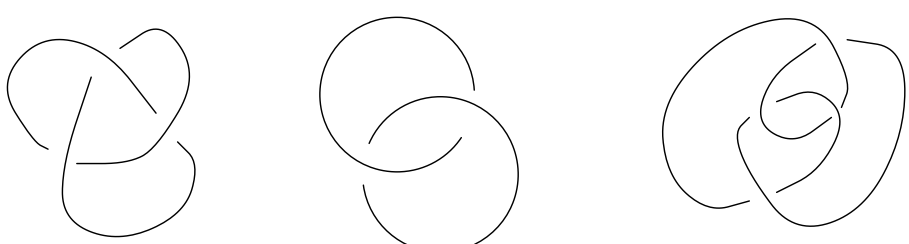
(i) **(ii)** **(iii)** **(iv)**

(ii) Use your results in (i) to calculate the linking numbers of the following 2-component links.



(i) **(ii)** **(iii)** **(iv)**

6. Draw all the states of the following knots and links. In each case, calculate the generalised bracket $\langle K \rangle$ from the states model, and hence evaluate the Kauffman bracket in each case. Finally, consider orientations of the diagram, and calculate the Jones polynomial of each oriented diagram from the Kauffman bracket and the writhe.



(i) **(ii)** **(iii)**