MT3522 Knot Theory Examples 4 2004/2005

1. Draw diagrams of the following braids:

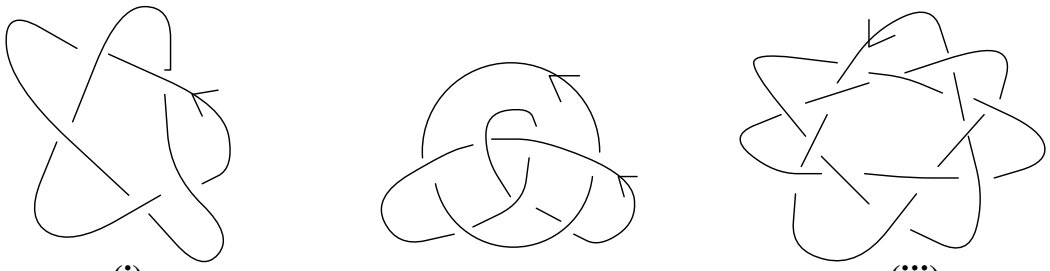
   (i) σ_1^2 (on two strings)

   (ii) $\sigma_1^{-1}\sigma_2\sigma_1\sigma_2^{-1}$ (on three strings)

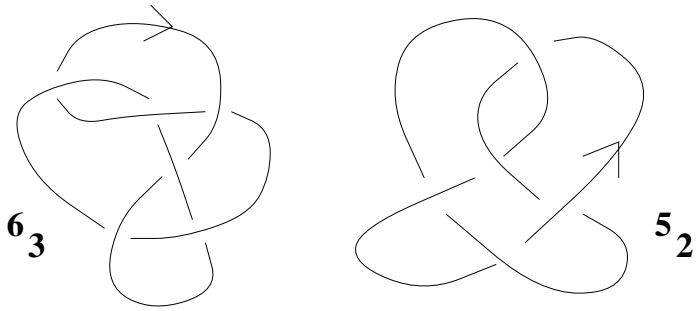
   (iii) $(\sigma_1^{-1}\sigma_2)^3$ (on three strings)

   Which knots or links are represented by the closures of these braids?

2. Describe each of the following knots or links as a closed braid by choosing a suitable centre.



3. Find closed braid representations of the following knots.



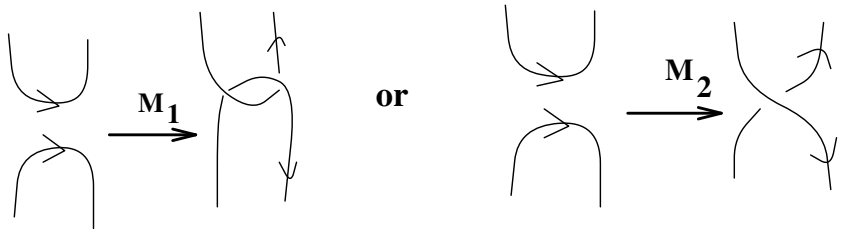
4. **(a)** The left hand trefoil knot is the closure of the braid σ_1^3 on two strings. Show that it is also the closure of the braids $\sigma_1^3\sigma_2$ and $\sigma_1^3\sigma_2^{-1}$ on three strings.

   (b) The figure eight knot is the closure of the braid $(\sigma_1\sigma_2^{-1})^2$ on three strings. Show that it is also the closure of each of the following braids on three strings:

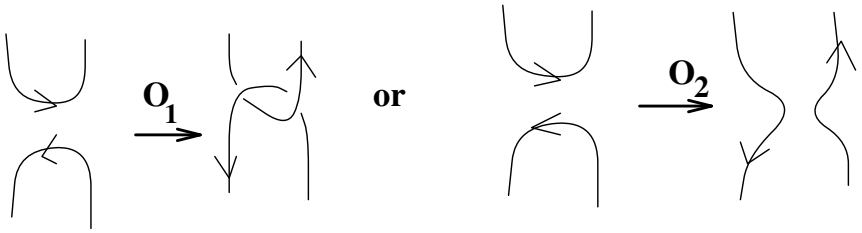
   (i) $(\sigma_2^{-1}\sigma_1)^2$,

   (ii) $\sigma_1^2\sigma_2^{-1}\sigma_1\sigma_2^{-1}\sigma_1^{-1}$.

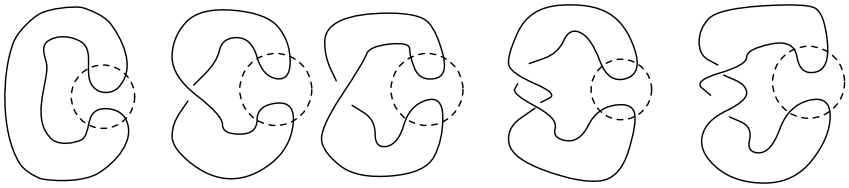
5. The action of a certain enzyme on single stranded DNA is known to be either



in matched occurrences of the pattern, and either



in opposite occurrences of the pattern. This gives four possible actions, namely (M_1, O_1), (M_1, O_2), (M_2, O_1),(M_2, O_2). A vat of cyclic single stranded DNA is prepared and the enzyme set to work. Assume that the substrate occurs as



in equal amounts, and that the amount with more twists is negligible. Let $n(T)$ be the number of trefoil knotted molecules in the product, and let $n(H)$ be the number of Hopf linked molecules in the product. The ratio $n(T) : n(H)$ is measured and found to be $1 : 2$.

Does this determine the action of the enzyme? Which actions of the enzyme could have given rise to this result?