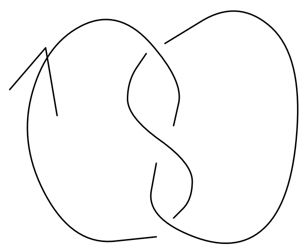
MT3522 Knot Theory Examples 3 2004/2005

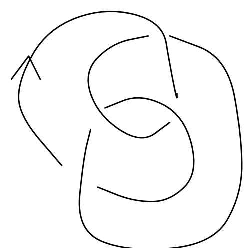
1. Show that (i) for the negative Hopf link H^-, $\nabla_{H^-} = -z$, and (ii) for the negative trefoil knot T^-, $\nabla_{T^-} = 1 + z^2$. (Thus the Conway polynomial distinguishes between the two Hopf links, but does *not* distinguish between the two trefoil knots.)

2. Complete the proof of Proposition 3.9, by showing that $a_1(L) = 0$ if L is a link with more than two components.

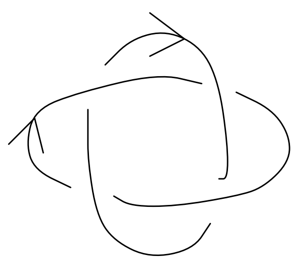
3. Calculate the Conway polynomial of each of the following knots K and links L by recursion using the three axioms.



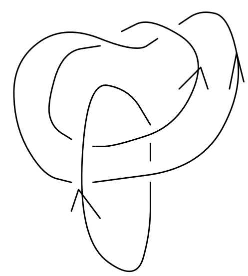
(i)



(ii)

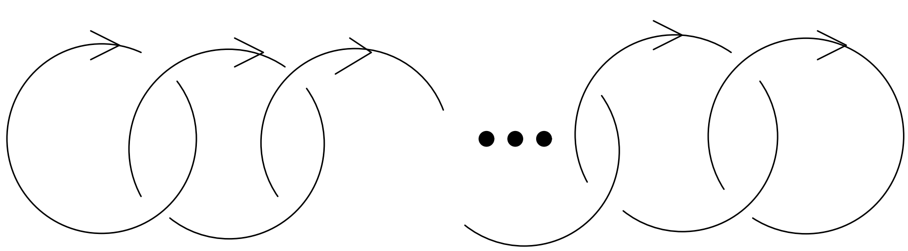


(iii)

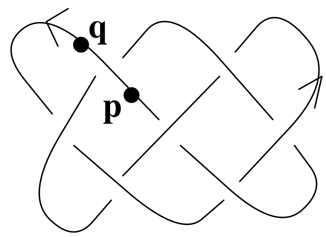


(iv)

4. Use induction on n to calculate the Conway polynomial of the n-link negative chain $L(n)$ shown below.

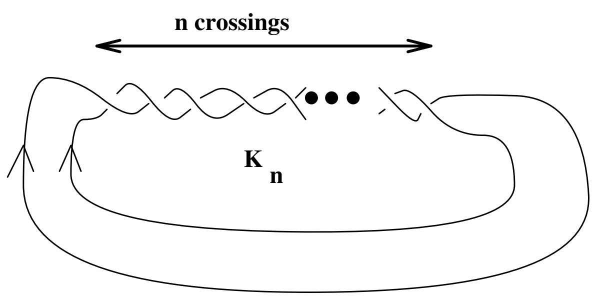


5. **(i)** Find the standard unknotting sequence for the following knot diagram K (the 7_4 knot), with the base point p. Repeat for the base point q. For each base point, calculate the function $\alpha(K)$ using Definition 3.11, and verify that $\alpha(K;p) = \alpha(K;q)$.



(ii) Repeat these calculations with the first two crossings c_1, c_2 in the standard unknotting sequence interchanged, and verify that the function $\alpha(K)$ remains unchanged.

6. Show that the diagram



where there are n crossings, represents a knot if n is odd and represents a two component link if n is even. Calculate $\nabla_n(z) = \nabla_{K_n}(z)$ for $n = 1$, 2 and 3. Prove the recursion relation

$$\nabla_n(z) = z\nabla_{n-1}(z) + \nabla_{n-2}(z).$$

Deduce that $\nabla_n(1)$ is the nth term in the Fibonacci series $1, 1, 2, 3, 5, 8, \ldots$, where each term is the sum of the two preceding numbers. Hence show that the knots and links K_n are all inequivalent.