1. Express the monomial symmetric functions

$$m_{(2,2,1)} = x^2y^2z + x^2yz^2 + xy^2z^2, \quad m_{(3,1,1)} = x^3yz + xy^3z + xyz^3$$

as polynomials in the elementary symmetric functions e_1, e_2, e_3.

2. Verify that each of the following polynomials is symmetric in x, y, z, and express it as a polynomial in e_1, e_2, e_3:

(i) $(x + y)(x + z)(y + z)$,
(ii) $(x + y)^3 + (x + z)^3 + (y + z)^3$,
(iii) $(x^2 + y^2 + z^2)^2$.

3. Given that the polynomial $x^3 + ax^2 + bx + c$ has nonzero roots α, β, γ, find the polynomial whose roots are

(i) $\alpha^2, \beta^2, \gamma^2$,
(ii) $\alpha^{-1}, \beta^{-1}, \gamma^{-1}$,

expressing the coefficients in terms of a, b, c.

4. Let α, β and γ be the three complex roots of the equation

$$x^3 + x + 1 = 0.$$

Calculate $\alpha + \beta + \gamma$, $\alpha^2 + \beta^2 + \gamma^2$, $\alpha^3 + \beta^3 + \gamma^3$ and $\alpha^4 + \beta^4 + \gamma^4$.

5. Let α, β and γ be three complex numbers satisying the equations

$$\begin{aligned} \alpha + \beta + \gamma &= 1, \\ \alpha^2 + \beta^2 + \gamma^2 &= 3, \\ \alpha^3 + \beta^3 + \gamma^3 &= 4. \end{aligned}$$

Find the polynomial whose roots are α, β and γ. Hence evaluate $\alpha^4+\beta^4+\gamma^4$ and $\alpha^5 + \beta^5 + \gamma^5$.

(Compare Examples 6, Question 2.)

6. Prove the identities

$$rh_r = \sum_{k=1}^{r} p_k h_{r-k}$$

relating the complete and power sum symmetric functions.