

Reminder: The class test will be held at 1.00 pm on Friday 9 March.

The material for the test is Sections 1–10, pages 1 to 26 of the course notes. The questions will be similar to questions on Example Sheets 1–4. There will be no bookwork questions, but you are advised to revise all definitions and statements of theorems etc, as you will not be allowed to refer to your notes during the test.

The test will contribute 20% of the course assessment.

1. Working over \mathbf{Q} , with Lex order and $x > y > z$, evaluate $S(g_1, g_2)$ for

(i) $g_1 = 3x^2y - xy^2$, $g_2 = 2xyz - 1$;

(ii) $g_1 = x + 1$, $g_2 = y - 1$;

(iii) $g_1 = x^4y^2z - 2x^2y^3z$, $g_2 = 2x^2yz^3 - 4y^2z^3$.

Deduce that g_1 and g_2 form a Gröbner basis for the ideal I that they generate in cases (ii) and (iii), but not in case (i).

2. In case (iii) above, show that xyz and y^2z^3 are not in I .

By computing remainders on division by g_1 and g_2 , decide which of $h = x^6y^6z^6 - 8y^9z^6$, $k = x^5y^2z^3$ is in I .

3. Find Gröbner bases for the following ideals in $\mathbf{Q}[x, y, z]$ with respect to Lex order with $x > y > z$.

(i) $I = \langle x^2y - y^2, x^2z - z^2 \rangle$;

(ii) $J = \langle x^2 - xz^2, y^2 - yz^3, z^3 - z \rangle$.