1. **(i)** Let F be a field. Write out an ordered list of monomials of degree 3 in $F[x, y, z]$ using the Lex, DegLex and DegRevLex orderings with $x > y > z$.

   (ii) Explain why the DegLex and DegRevLex orderings on monomials in $F[x, y]$ with $x > y$ are the same.

   (iii) Invent a monomial ordering on $F[x, y]$ which is different from the examples given in the notes.

2. Let F be a field and let $<$ be a monomial ordering on $P = F[x_1, \ldots, x_n]$.

   (i) Let $f \in P$, $f \neq 0$, and let m be a monomial. Show that $\mathrm{LT}(m \cdot f) = m \cdot \mathrm{LT}(f)$.

   (ii) Prove that the leading term of the product $f \cdot g$ of two nonzero polynomials f, g in P is the product of their leading terms. Show by means of an example that the corresponding statement for the sum $f + g$ is false in general.

   (iii) Let $f_i, g_i \in P$, $1 \leq i \leq s$. Can we always find a value of k such that

   $$\mathrm{LM}\left(\sum_{i=1}^{s} f_i \cdot g_i\right) = \mathrm{LM}(f_k) \cdot \mathrm{LM}(g_k)?$$

3. For each of the following monomial orders in $\mathbf{Q}[x, y, z]$

   (a) Lex order with $x > y > z$,

   (b) Lex order with $z > y > x$,

   (c) DegLex order with $x > y > z$,

   write f in standard form and give $\mathrm{LM}(f)$, $\mathrm{LC}(f)$ and $\mathrm{LT}(f)$, where

   (i) $f = 2x^3y^2z - xy^4z + 4x^2y^2z^2$,

   (ii) $f = xy - 3x^2 - xy^2 + z^3$.

4. For Lex order in $\mathbf{Q}[x, y, z]$ with $x > y > z$, find the remainder of f on division by g, and write it in standard form:

   (i) $f = x^3 - x^2y + 2y^3$, $g = x^2 - xy$;

   (ii) $f = 4x^3z - z^2$, $g = 2x^2 - xy$.

   (iii) Taking $h = yz - z + 1$ in **(ii)**, find the quotients q_1 and q_2 and the remainder r of f on division by the (ordered) pair of polynomials (g, h).