Optimal Control with Partial Information for Stochastic Volterra Equations

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Abstract

In the first part of the paper, we obtain existence and characterizations of an optimal control for a linear quadratic control problem of linear stochastic Volterra equations. In the second part, using the Malliavin calculus approach, we deduce a general maximum principle for optimal control of general stochastic Volterra equations. The result is applied to solve some stochastic control problem for some stochastic delay equations.

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1 Introduction

Let $(\Omega, \mathcal{F}, \mathcal{F}_t, P)$ be a filtered probability space and $B(t), t \geq 0$ a $\mathcal{F}_t-$ real valued Brownian motion. Let $R_0 = R \setminus \{0\}$ and $\nu(dz)$ a $\sigma$-finite measure on $(R_0, B(R_0))$. Let $N(dt, dz)$ denote a stationary Poisson random measure on $R_+ \times R_0$ with intensity measure $d\nu(dz)$. Denote by $\tilde{N}(dt, dz) = N(dt, dz) - dt\nu(dz)$ the compensated Poisson measure. Suppose we have a cash flow where the amount $X(t)$ at time $t$ is modelled by a stochastic delay equation of the form:

$$dX(t) = \{A_1(t)X(t) + A_2(t)X(t-h) + \int_{t-h}^t A_0(t,s)X(s)ds\}dt$$

$$+ C_1(t)dB(t) + \int_{R_0} C_2(t,z)\tilde{N}(dt,dz); \ t \geq 0$$

$$X(t) = \eta(t); \ t \in [-h,0].$$

Here $h > 0$ is a fixed delay and $A_1(t), A_2(t), A_0(t,s), C_1(t), C_2(t,z), \eta$ are given bounded deterministic functions.

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