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ACTIVE CONTROL OF FRICTION-DRIVEN OSCILLATIONS

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This paper introduces an active control technique that combats oscillations driven by dry friction forces. Dry friction can act as the excitation mechanism for some high amplitude oscillations; curve squeal from trains is a well known and notorious example. Another, more elementary, example is the oscillation of a mass–spring system sliding on a moving belt. A model which predicts the stability behaviour of this system is presented. The model is then extended to include an active control system. The active control system is a feedback loop which consists of the following components. A transducer senses the velocity of the mass. The transducer output is passed to a filter, then to a phase shifter which applies an adjustable phase-shift, to a variable-gain amplifier, and finally to a shaker which is attached to the mass. The shaker exerts a control force in the tangential direction, and this force superimposes on the friction force. The stability behaviour of the controlled system is analyzed in two ways: by calculation of the complex eigenfrequencies and by considerations based on a balance equation for the oscillatory energy. The differences between this form of active control and conventional anti-vibration methods are discussed. Further studies, both on the theoretical and experimental front, are under way to extend the application of this kind of active control to curve squeal from trains.

1. INTRODUCTION

The aim of this paper is to introduce an active control technique that combats oscillations driven by dry friction forces. Dry friction forces provide the excitation mechanism for some high amplitude oscillations, such as those of a squealing railway wheel or of a vibrating string excited by a bow. Dry friction forces act tangentially at the contact area between two bodies. If the two bodies slide relative to each other, i.e., if they are in a state of "slipping", the dry friction force acts as a resistance against this relative motion. In the "sticking" state, exceedingly small departures from zero relative motion are resisted by relatively large frictional forces. Stick and slip can occur successively in a system capable of oscillation and drive a periodic motion. Such oscillations may often be observed indirectly by the sound they radiate into the surrounding air.

A very simple system, in which stick–slip oscillations can be observed directly, consists of a mass–spring system on a belt moving with constant speed (see Figure 1). This system will serve as an elementary example of friction-driven oscillations for the purposes of this paper. A mathematical model will be developed to predict the stability behaviour of this system. The model will then be extended to include the effect of a feedback system which exerts a control force, synchronized with the oscillation, on the mass. The control force is applied with a shaker driven by a signal that is a phase-shifted and amplified version of the mass velocity.

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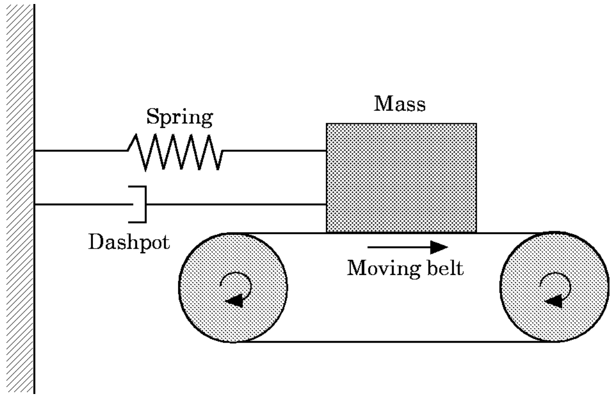

Figure 1. A damped mass–spring system on a moving belt.

This form of active control is analogous to the method used by Heckl [1] to control a heat-driven sound field in a tube. It differs from conventional anti-sound and anti-vibration techniques, which combat noise or vibrations by superposition with a signal in exact anti-phase. The features distinguishing conventional techniques from our technique will be discussed.

2. MATHEMATICAL MODEL

2.1. DRY FRICTION CHARACTERISTIC

Dry friction is not easy to model. It depends on a number of parameters, such as the relative velocity of the contacting bodies, normal contact force, surface properties and material properties. Dry friction forces that show a stick–slip behaviour have been modelled by several authors, e.g., McIntyre and Woodhouse [2], who studied the dynamics of a bowed string. In their model, the friction force F_f depends on the relative velocity $v - V$ (where v is the oscillation velocity of the string, and V is the speed of the bow) and is proportional to the normal force. The functional dependence $F_f(v)$, which is called the "friction characteristic", is shown in Figure 2. Points on the very steep section of the friction characteristic, where v is close to V, correspond to sticking. Points on the left of the maximum represent states of slipping. The positive slope of the slip section of this friction characteristic is an important feature in that it makes unstable oscillations possible.

The emphasis in this paper is on the *control* of friction-driven oscillations, rather than on the detailed properties of dry friction forces. We therefore assume, without further justification, that this friction characteristic is valid also for the belt-driven mass. V denotes the belt speed and v is the velocity of the oscillating mass. For small oscillation amplitudes, where the velocity does not reach the stick section of the friction characteristic, the friction characteristic can be approximated by a linear function (the dashed line in Figure 2). This linear approximation will be sufficient for the prediction of the stability behaviour. Only a study of unstable oscillations and their behaviour at high amplitudes requires a non-linear treatment. The friction force is modelled by

$$F_f = F_0 + \gamma v. \tag{2.1}$$

γ denotes the (positive) slope of the curve and F_0 is the friction force at zero velocity, $v = 0$.

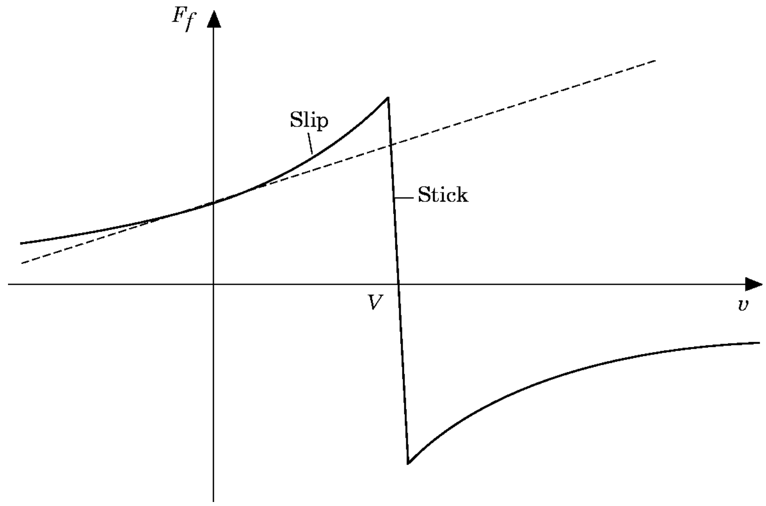

Figure 2. The friction characteristic used by McIntyre and Woodhouse (solid line) and its linearized form (dashed line).

2.2. BELT-DRIVEN MASS–SPRING OSCILLATOR

The subject of this study is a damped mass–spring oscillator on a moving belt (see Figure 1). The oscillator is excited by friction processes in the interface between the mass and the belt. The mass is m, the spring has stiffness k, and the damping coefficient of the dashpot is c. The system is governed by the equation of motion

$$m(\partial^2 w/\partial t^2) + c(\partial w/\partial t) + kw = F(v), \tag{2.2}$$

where w is the displacement of the mass, t is time and F is the excitation force. The excitation force considered here is the friction force given by equation (2.1), and so equation (2.2) becomes

$$m(\partial^2 w/\partial t^2) + (c - \gamma)\partial w/\partial t + kw = F_0. \tag{2.3}$$

One can clearly see here the role of γ, the (positive) slope of the linearized friction characteristic: $\gamma > 0$ guarantees that the friction force has the effect of "negative damping". The solution of equation (2.3) is of the form

$$w(t) = a\,\mathrm{e}^{-\mathrm{i}\Omega t} + b, \tag{2.4}$$

where Ω is the complex eigenfrequency of the belt-driven oscillator; and a and b are constants. Ω and b can be determined by insertion of equation (2.4) into equation (2.3), and one obtains

$$b = F_0/k, \qquad \Omega = \pm\sqrt{(k/m) - (1/4m^2)(\gamma - c)^2} + \mathrm{i}(\gamma - c)/2m. \tag{2.5a, b}$$

The real and imaginary parts of Ω,

$$\Omega = \omega + \mathrm{i}\delta, \tag{2.6}$$

are physically meaningful. The real part ω is the frequency of the friction-driven oscillation. It is close to the resonance frequency $\omega_0 = \sqrt{k/m}$ of the free mass–spring oscillator if $(\gamma - c)^2 \ll mk$: i.e. if the net damping of the system is small. The imaginary part δ is the growth rate of the oscillation. The sign of δ provides a criterion for the stability behaviour of the friction-driven oscillator,

$$\left\{\begin{array}{l}\text{unstable}\\ \text{critically stable}\\ \text{stable}\end{array}\right\} \quad \text{if } \delta\left\{\begin{array}{c}>\\=\\<\end{array}\right\}0, \tag{2.7}$$

It is clear from equation (2.5b) that the oscillation is unstable with an exponentially growing amplitude, if

$$\gamma > c; \tag{2.8}$$

this is the case if the slope of the friction characteristic exceeds the damping.

The second solution in equation (2.5b) for Ω with the negative real part is of no physical interest and is therefore ignored.

2.3. ACTIVE CONTROL

The model is now extended to include the active control system shown in Figure 3. The control system consists of a feedback loop with the following components. A transducer senses the velocity of the mass. The transducer output is passed to a filter which separates the oscillation frequency from any potentially contaminating signal. A phase-shifter applies an adjustable phase shift to the filtered signal. The phase-shifted signal is passed to a variable gain amplifier to be amplified by an appropriate amount. A shaker, which is attached to the mass, excites tangential oscillations which superimpose on the oscillations caused by the friction force.

The force exerted by the shaker is called the "control force" and denoted by F_c. It is a phase-shifted and amplified version of the oscillation velocity v and can be described mathematically by

$$F_c = \alpha\, e^{-i\phi} v; \tag{2.9}$$

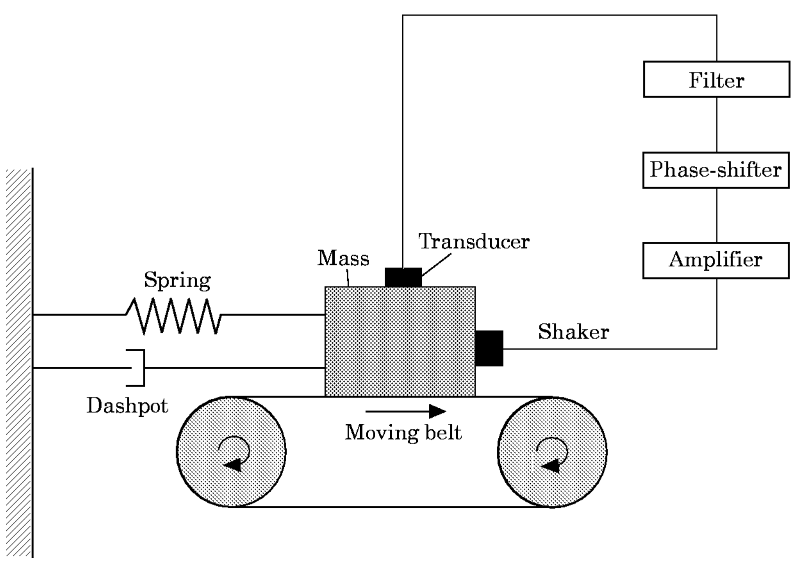

Figure 3. Active control of the belt-driven oscillator.

α is a measure for the amplification and ϕ is the phase shift. When F_c is added to the friction force, the equation of motion becomes

$$m(\partial^2 w/\partial t^2) + (c - \gamma)\partial w/\partial t + kw = F_0 + \alpha\, e^{-i\phi}\partial w/\partial t. \tag{2.10}$$

The complex eigenfrequency Ω of the controlled belt-driven oscillator is determined in the same way as in section 2.2: i.e., by assuming a solution of the form (2.4) and inserting it in the equation of motion (2.10). One obtains the quadratic equation

$$m\Omega^2 + i\Omega(c - \gamma - \alpha\, e^{-i\phi}) - k = 0, \tag{2.11a}$$

which has the solution

$$\Omega = \sqrt{(k/m) - (1/4m^2)(\gamma - c + \alpha\, e^{-i\phi})^2} + i(\gamma - c + \alpha\, e^{-i\phi})/2m. \tag{2.11b}$$

This result gives the frequency and growth rate of the controlled oscillation as a function of the control parameters α and ϕ. In the absence of control, i.e., if $\alpha = 0$, the eigenfrequency reduces to the expression (2.5b). It is clear that α and ϕ have an influence not just on the real part, but also on the imaginary part of the eigenfrequency and can even change the sign of the imaginary part to give a negative growth rate. If this happens, control is achieved, and a previously unstable oscillation is made to decay exponentially.

It is of interest to find the values of α and ϕ which give a particular growth rate. For this purpose, α and ϕ are written as explicit functions of ω and δ. After a few straightforward manipulations, starting from equation (2.11a), one obtains

$$\alpha = \sqrt{R^2 + I^2}, \qquad \phi = \arg(-I/R) \tag{2.12a, b}$$

with

$$R = \delta\,\frac{(\omega^2 + \delta^2)m + k}{\omega^2 + \delta^2} + c - \gamma, \qquad I = \omega\,\frac{-(\omega^2 + \delta^2)m + k}{\omega^2 + \delta^2}. \tag{2.13a, b}$$

These equations can be used to plot curves $\delta =$ constant and $\omega =$ constant in the α–ϕ plane. Of particular interest is the curve $\delta = 0$ because this is the demarcation line between stability and instability for the case of a control system which has only a phase-shifter and an amplifier, but no filter.

No attention has been paid so far to the way in which the oscillation frequency ω is influenced by the control parameters α and ϕ. However, this is an important question when choosing the frequency band passed by the filter of the control system to the phase-shifter and amplifier, unless the oscillation frequency lies within this band, the control will not be effective. Maximizing the filter bandwidth increases the risk of contamination by noise, and is therefore not a practical solution. A suitable filter setting, comprising a narrow frequency band which includes the oscillation frequency ω, can be chosen relatively easily in practice if the values of α and ϕ have little effect on ω. If ω_{min} and ω_{max} are the minimum and maximum frequencies of the passed frequency band, then the area in the α–ϕ plane between the curves $\omega = \omega_{min}$ and $\omega = \omega_{max}$ indicates the stability range imposed by the filter setting.

3. NUMERICAL RESULTS

Equations (2.12a) and (2.12b) can be evaluated numerically to give results in the form of curves $\delta =$ constant and $\omega =$ constant in the α–ϕ plane. For the results presented here, the quantity $\gamma - c$, which is a measure for the strength of the excitation of the uncontrolled system, was set the same for all cases, $\gamma - c = 0{\cdot}3\sqrt{mk}$; this represents quite a strong excitation.

The curve $\delta = 0$ in the α–ϕ plane is shown in Figure 4(a); the amplification values along the α-axis are given as multiples of $\sqrt{mk}$. Without any frequency restrictions, the stability region in the α–ϕ plane is rather large. The optimum phase-shift is at 180°, achieving stability if $\alpha > 0{\cdot}3\sqrt{mk} = \gamma - c$. The oscillator becomes stable for any ϕ between 90° and 270°, so long as α is sufficiently large. Also shown in this figure are the curves $\delta = -0{\cdot}1\omega_0$ and $\delta = -0{\cdot}2\omega_0$ (ω_0 is the resonance frequency of the freely oscillating mass/spring system). Both of these values for δ represent stable cases: within one oscillation period,

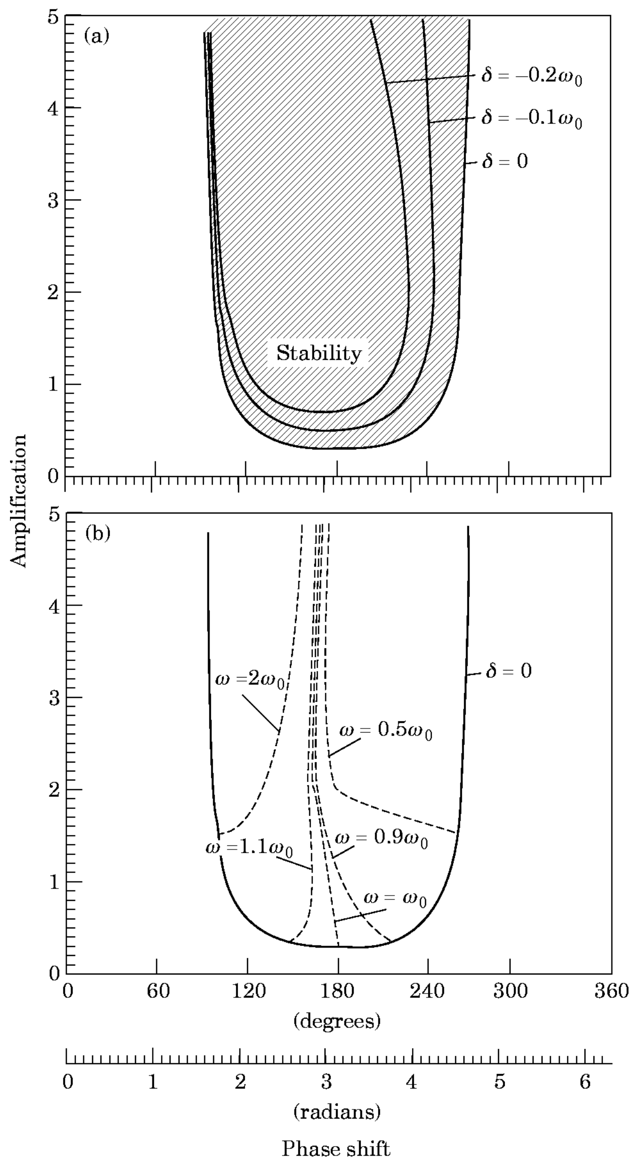

Figure 4. Curves of constant δ and constant ω in the α–ϕ plane. The amplification is given in non-dimensional form, as multiples of $\sqrt{mk}$; ω_0 is the resonance frequency of the free oscillator. (a) curves $\delta = 0$, $\delta = -0{\cdot}1\omega_0$, $\delta = -0{\cdot}2\omega_0$; (b) curves $\delta = 0$ (solid line) and $\omega = 0{\cdot}5\omega_0$, $\omega = 0{\cdot}9\omega_0$, $\omega = \omega_0$, $\omega = 1{\cdot}1\omega_0$, $\omega = 2\omega_0$ (dashed lines).

the amplitude decays by a factor 0·5 and 0·3 respectively for $\delta = -0{\cdot}1\omega_0$ and $\delta = -0{\cdot}2\omega_0$, leading to a virtual extinction of the oscillation within just a few periods.

The need to frequency-filter the control signal reduces the size of the stability region. The reduction depends on the filter bandwidth. Various curves of constant frequency in the α–ϕ plane are shown in Figure 4(b). The middle curve is $\omega = \omega_0$; the outer curves are for frequency values $\omega = 0{\cdot}9\omega_0$ and $1{\cdot}1\omega_0$ (10% below and above the resonance frequency), and for $\omega = 0{\cdot}5\omega_0$ and $2\omega_0$ (a factor of 2 below and above the resonance frequency).

For practical purposes, it is best to choose the control parameters α and ϕ from the lower end of the stability region. The range of phase values which give stability is then greater. Even when the filter has been set to let through only a narrow frequency band of $\pm 10\%$ of ω_0, there is a 65° wide phase range within which control is achieved. This phase range becomes narrower with increasing amplification α. One might be tempted to operate with a large value for α because if α is increased, the controlled oscillation decays more rapidly (see Figure 4(a)). However, even with a small α, just above the curve $\delta = 0$, significant decay rates are achieved by the control system, so there is little need to maximize α with the aim of getting a rapid enough amplitude decay.

4. ENERGY CONSIDERATIONS

While the calculation of the complex eigenfrequencies gives all the information about the stability behaviour of the system, it does not give much physical insight into the active control. In this section, some light will be shed on the physics of the active control, and in particular on the question "What happens to the energy?". The balance equation for the oscillatory energy of the mass–spring system will form the basis of these considerations.

An energy balance equation for the oscillator can be derived from the equation of motion (2.2). After multiplication of both sides by $\partial w/\partial t$ (real notation is now used instead of complex notation) and rewriting products of time derivatives, the result can be brought into the form

$$(\partial/\partial t)(\tfrac{1}{2}mv^2 + \tfrac{1}{2}kw^2) = vF - cv^2. \tag{4.1}$$

This represents the required balance equation. The term in brackets is the sum of the kinetic and potential energy, so the left side of equation (4.1) is the rate of change of the total energy of the oscillator. This energy change is brought about by two effects: the external force (first term on the right side) and the damping (second term on the right side). The first term represents the energy change induced by the external force. This can be positive or negative, depending on whether force and velocity have the same or opposite signs. If they have the same sign, the force accelerates the mass in the direction of its velocity, thus adding to the velocity. If they have opposite signs, the mass is decelerated.

In an oscillatory motion, the product vF generally fluctuates, and it is the average $\overline{vF}$ (averaged over one period of the oscillation) that gives an indication of the long-term energy change. The second term $-cv^2$ is always negative (or zero) and so is its average; it represents the energy lost in the dashpot. A time-average of the energy balance (4.1) can be used as a criterion for instability. The energy, and thus the amplitude of the oscillation, increases from one cycle to the next if the averaged energy change

$$\overline{\Delta E} = \overline{vF} - \overline{cv^2} \tag{4.2}$$

is positive, whereas there is stability if the net energy gain is negative:

$$\begin{Bmatrix} \text{unstable} \\ \text{critically stable} \\ \text{stable} \end{Bmatrix} \qquad \text{if } \overline{\Delta E} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$$

The behaviour of $\overline{\Delta E}$ is now analyzed for a mass–spring system excited by an external force which is a superposition of the friction force F_f (see equation (2.1)) and the control force F_c (see equation (2.9) for F_c in terms of a phase shift ϕ); the control force is written in terms of a time lag τ, as $F_c = \alpha v(t-\tau)$, to comply with real notation. One obtains

$$\overline{\Delta E} = \overline{v(F_f + F_c)} - \overline{cv^2} = \overline{v(t)F_0} + \gamma\overline{v^2(t)} + \alpha\overline{v(t)v(t-\tau)} - \overline{cv^2(t)}. \qquad (4.3)$$

The first term on the right side is zero. The second term represents the energy gain from the "positive damping" by the friction force. The third term represents the energy change by the control system. The fourth term is the energy lost due to the damping of the dashpot.

The control term can change the overall balance. If $\overline{\Delta E} > 0$ in the uncontrolled case, application of control can make $\overline{\Delta E}$ negative, provided that the time lag τ (or equivalently the phase shift ϕ) has been chosen such that $\overline{v(t)v(t-\tau)} < 0$, and provided that the amplification α is large enough. These two requirements are reflected in Figure 4(a). A phase shift of 180° is optimal, but a large range of phases around 180° will still give control. A phase shift of 180° is equivalent to a time lag of half a period, because then $\overline{v(t)v(t-\tau)} = -\overline{v^2(t)}$. The amplification α has to be above some critical value which increases with increasing deviation from the optimal phase shift. At the optimal phase shift of 180°, α has to satisfy $\alpha > \gamma - c$ in order to achieve stability: i.e., $\overline{\Delta E} < 0$.

Another interesting effect can be seen from equation (4.3). The energy change brought about by the control system is $\alpha\,\overline{v(t)v(t-\tau)}$; it depends on the velocity amplitude. If control is applied successfully, the velocity amplitude decays and thus the energy change induced by the control system decays, too. There is therefore no need to maintain the full anti-vibration power throughout the control.

5. CONCLUSIONS AND OUTLOOK

Friction-driven oscillations of a simple mass–spring oscillator can be suppressed by a feedback system consisting of a sensor, frequency filter, phase-shifter, amplifier and a shaker attached to the mass. It is of interest to compare the features of conventional anti-vibration techniques with those of our feedback system. (i) Anti-vibration techniques do not affect the vibration generation mechanism; they work by superposition of a secondary vibration on a primary one in such a way that the combined vibration is weaker than the primary one. (ii) It is necessary to maintain the full anti-vibration power throughout the control. (iii) The effectiveness of the control is very sensitive to errors in phase shift and amplification; even small errors of just a few degrees in phase shift and a few percent in amplification will lead to ineffective control.

In contrast to these points, our feedback system has the following features. (i) It works specifically for unstable vibrations driven by dry friction; it suppresses an unstable vibration by interfering with the vibration generation mechanism. (ii) The power supplied by the controller decreases with the vibration amplitude; once control has been achieved, very little power is required. (iii) The effectiveness of the control is very insensitive to errors in phase shift and amplification; control can be achieved within a phase range of 65° if the amplification is not too small.

Our active control method represents a promising technique for the control of other friction-driven oscillations, in particular squeal noise, which is a particularly disturbing type of noise. There are many parallels between the generation of curve squeal from trains and the friction-driven mass–spring oscillator considered in this paper. Curve squeal occurs when there is a lateral progression of the wheel relative to the rail. This leads to dry friction forces with a lateral component which excite bending vibrations of the wheel. These vibrations are radiated into the surrounding air and heard as intense high pitched squeal. Squeal noise has a discrete spectrum of frequencies; these are the frequencies of the different modes of the wheel.

In some cases, the spectrum is dominated by a single frequency; one can then consider the wheel as a lightly damped oscillator with a given resonance frequency and apply the active control described in this paper. If there are several peaks in the frequency spectrum, it may be necessary to control each of these individually. The frequency peaks could be separated by a filter, and a phase shift and amplification could be applied to each frequency individually.

An active control technique, analogous to that described in section 2, has been developed with a small scale rig simulating curve squeal. The wheel is modelled by a flat circular disc. Its edge is in contact with a flat surface, and there is a relative movement in the lateral direction. The squeal from this particular rig has only one frequency peak in its spectrum. The feedback loop is shown in Figure 5; it consists of the following components. A microphone senses the squeal noise. The microphone signal is fed into a variable phase-shifter with a narrow-band filter that only passes the signal frequency. A variable gain amplifier amplifies the signal from the phase-shifter. A piezoelectric film, attached close to the contact area, receives the signal and excites bending vibrations of the disc. In preliminary experiments some control has been achieved. Further experiments are in progress.

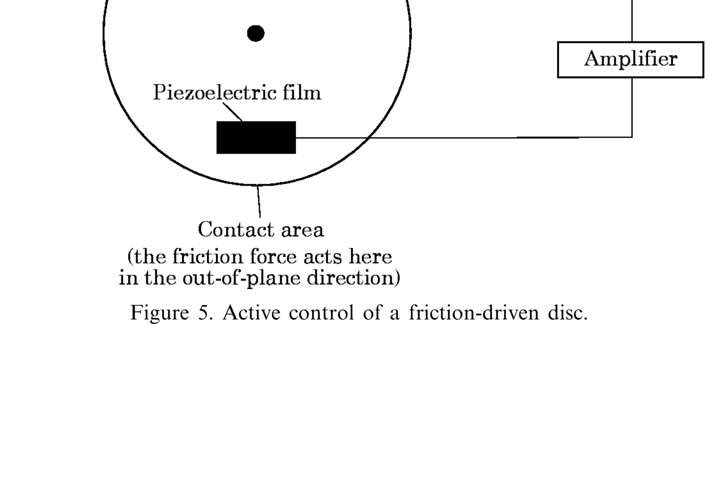

Figure 5. Active control of a friction-driven disc.

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