VECTOR FORMULAE

Scalar product $\mathbf{a}.\mathbf{b} = ab\cos\theta = a_1b_1 + a_2b_2 + a_3b_3$

Vector product $\mathbf{a}\times\mathbf{b} = ab\sin\theta\hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

$$= (a_2b_3 - a_3b_2)\mathbf{i} + (a_3b_1 - a_1b_3)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}$$

Triple products

$$[\mathbf{a},\mathbf{b},\mathbf{c}] = (\mathbf{a}\times\mathbf{b}).\mathbf{c} = \mathbf{a}.(\mathbf{b}\times\mathbf{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}.\mathbf{c})\mathbf{b} - (\mathbf{a}.\mathbf{b})\mathbf{c}$$

Vector Calculus

$$\nabla \ \equiv \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

grad $\phi \equiv \nabla\phi$, div $\mathbf{A} \equiv \nabla.\mathbf{A}$, curl $\mathbf{A} \equiv \nabla\times\mathbf{A}$
div grad $\phi \equiv \nabla.(\nabla\ \phi) \equiv \nabla^2\phi$ (for scalars only)
div curl $\mathbf{A} = 0$ curl grad $\phi \equiv \mathbf{0}$
$\nabla^2\mathbf{A} =$ grad div $\mathbf{A} -$ curl curl $\mathbf{A}$
$\nabla(\alpha\beta) = \alpha\,\nabla\beta + \beta\,\nabla\alpha$
div $(\alpha\mathbf{A}) = \alpha$ div $\mathbf{A} + \mathbf{A}.(\nabla\alpha)$
curl $(\alpha\mathbf{A}) = \alpha$ curl $\mathbf{A} - \mathbf{A}\times(\nabla\alpha)$
div $(\mathbf{A}\times\mathbf{B}) = \mathbf{B}.$ curl $\mathbf{A} - \mathbf{A}.$ curl $\mathbf{B}$
curl $(\mathbf{A}\times\mathbf{B}) = \mathbf{A}$ div $\mathbf{B} - \mathbf{B}$ div $\mathbf{A} + (\mathbf{B}.\nabla\ )\mathbf{A} - (\mathbf{A}.\nabla\ )\mathbf{B}$
grad $(\mathbf{A}.\mathbf{B}) = \mathbf{A}\times$ curl $\mathbf{B} + \mathbf{B}\times$ curl $\mathbf{A} + (\mathbf{A}.\nabla\ )\mathbf{B} + (\mathbf{B}.\nabla\ )\mathbf{A}$

Integral Theorems

Divergence theorem

$$\int_{\text{surface}} \mathbf{A}.d\mathbf{S} = \int_{\text{volume}} \text{div } \mathbf{A}\ dV$$

Stokes' theorem

$$\int_{\text{surface}} (\text{ curl } \mathbf{A}).d\mathbf{S} = \oint_{\text{contour}} \mathbf{A}.d\mathbf{r}$$

Green's theorems

$$\int_{\text{volume}} (\psi\nabla^2\phi - \phi\nabla^2\psi)dV = \int_{\text{surface}} \left(\psi\frac{\partial\phi}{\partial n} - \phi\frac{\partial\psi}{\partial n}\right)|d\mathbf{S}|$$

$$\int_{\text{volume}} \left\{\psi\nabla^2\phi + (\nabla\phi)(\nabla\psi)\right\}dV = \int_{\text{surface}} \psi\frac{\partial\phi}{\partial n}|d\mathbf{S}|$$

where

$$d\mathbf{S} = \hat{\mathbf{n}}|d\mathbf{S}|$$

Green's theorem in the plane

$$\oint (Pdx + Qdy) = \int\int \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\ dy$$